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PROBABILISTIC EVALUATION METHOD OF LIVING ENVIRONMENTAL NOISE CONTROL SYSTEM BY USE OF MODIFIED STATISTICAL ENERGY ANALYSIS METHOD

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INTRODUCTION

In the practical engineering field of the actual noise control system, it is as important as the sound source countermeasure to improve the sound propagation characteristics by newly setting a sound insulation system or by reinforcing the present state of a sound insulation system. On the other hand, for the purpose of evaluating the transmitted noise level fluctuation of sound insulation system, it is especially necessary to grasp quantitatively the statistical characteristics of this noise fluctuation.

From the above practical point of view, in this paper, the acoustic transmission characteristics of sound insulation system such as single-wall and double-wall is first estimated by use of the usual and modified S.E.A. methods. Next, when a general stationary random noise of arbitrary distribution type is passed through various kinds of sound insulation systems , a new probabilistic evaluation method for the transmitted noise fluctuation is theoretically proposed from the following two points of view. As the first method, the explicit expression of the transmitted noise intensity distribution after system change is derived in a general form of statistical orthogonal and/or non-orthogonal expansion series, taking the transmitted noise intensity distribution function observed before the system change into the first expression term. As the second method, a statistical evaluation method of the transmitted noise fluctuation after improving the sound insulation system is given based on the information on several statistical moments of the incident noise fluctuation and the acoustic transmission characteristics of sound insulation system.

Finally, the validity of the proposed theoretical procedure is experimentally confirmed by applying to the actual transmitted noise data of sound insulation systems observed in the reverberation room.

THEORETICAL CONSIDERATIONS

For the purpose of evaluating various kinds of noise indices like L_x and L_{eq}, it is essentially necessary in this probabilistic evaluation to find out the explicit expression of probability distribution function of the transmitted noise intensity. In this section, three typical methods are proposed for the probabilistic evaluation of the transmitted noise.

Non-orthogonal expression of reflecting system change

Let x_ℓ ($\ell=1,2,\cdots,N$) be the energy component of noise intensity existing in the ℓth frequency band-width of the incident noise fluctuation x. Let y and z be respectively two output intensity fluctuations observed before and after improvement of the sound insulation system with the same input excitation x. And let b_ℓ and a_ℓ reflect respectively the acoustic transmission characteristics after and before improvement of the system. From the additive property of energy quantity, the overall transmitted noise intensity y and z are given as

$$y=\sum_{\ell=1}^{N} b_\ell x_\ell \quad , \quad z=\sum_{\ell=1}^{N} a_\ell x_\ell \quad . \tag{1}$$

In this case, the probability density function, $P_z(z)$, for the random noise intensity, z, can be expressed on the basis of probability density function, $P_y(y)$, as follows :

$$P_z(z)=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}A_n\frac{d^n}{dz^n}P_y(z) \tag{2}$$

with

$$\left.\begin{aligned} &A_0=1 \quad , \quad A_1=\sum_{\ell=1}^{N}(a_\ell-b_\ell)\langle x_\ell\rangle \quad , \\ &A_2=\sum_{\ell,k=1}^{N}(a_\ell a_k-b_\ell b_k)\langle(x_\ell-\langle x_\ell\rangle)(x_k-\langle x_k\rangle)\rangle+A_1^2,\cdots \quad . \end{aligned}\right\} \tag{3}$$

Especially when the statistical Hermite series type expression defined as

with $\mu_y=\langle y\rangle$, $\sigma_y^2=\langle(y-\mu_y)^2\rangle$,

$$P_y(z)=\frac{1}{\sqrt{2\pi}\sigma_y}e^{-\frac{(z-\mu_y)}{2\sigma_y^2}}\sum_{r=0}^{\infty}B_rH_r\left(\frac{z-\mu_y}{\sigma_y}\right) \quad (4) \qquad B_r\equiv\frac{1}{r!}\left\langle H_r\left(\frac{y-\mu_y}{\sigma_y}\right)\right\rangle \quad (5)$$

is employed as $P_y(z)$ of arbitrary distribution type for the output intensity before the system change, Eq.(2) can be explicitly expressed as

$$P_z(z)=\frac{1}{\sqrt{2\pi}\sigma_y}e^{-\frac{(z-\mu_y)}{2\sigma_y^2}}\sum_{n=0}^{\infty}\sum_{r=0}^{\infty}\frac{A_n}{n!}\frac{B_r}{\sigma_y^2}H_{n+r}\left(\frac{z-\mu_y}{\sigma_y}\right) \quad . \tag{6}$$

Orthogonal expression of reflecting system change

First, consider the probability density function $P_z(z)$ expressed in the statistical orthogonal expansion form :

$$P_z(z)=P_y(z)\sum_{n=0}^{\infty}C_n\theta_n(z) \quad , \quad C_n\equiv\langle\theta_n(z)\rangle \quad , \tag{7}$$

where $\theta_n(z)$ is the preestablished polynomial satisfying the following orthogonality relation with a weighting function $P_y(z)$:

$$\int_{-\infty}^{\infty}P_y(z)\theta_m(z)\theta_n(z)=\sigma_{mn} \quad (m,n=0,1,2,\cdots) \; . \tag{8}$$

That is, $\theta_n(z)$ can be concretely formed by the linear combination of

$\phi_i(z)$ as follows : $$\theta_n(z)=\sum_{j=0}^{n}\lambda_{nj}\phi_j(z)\ , \tag{9}$$
where λ_{nj} is given by the so-called Schmidt's orthogonalization procedure as
$$\lambda_{nj}=(d_{n-1}d_n)^{-1/2}\cdot((n,j)\text{cofactor of } d_n(z)) \tag{10}$$
$$d_n(z)=\begin{vmatrix}(\phi_0,\phi_0) & (\phi_0,\phi_1) & \cdots & (\phi_0,\phi_n)\\ (\phi_1,\phi_0) & (\phi_1,\phi_1) & \cdots & (\phi_1,\phi_n)\\ \cdots & & & \\ (\phi_{n1},\phi_0) & (\phi_{n1},\phi_1) & \cdots & (\phi_{n1},\phi_n)\\ \phi_0(z) & \phi_1(z) & \cdots & \phi_n(z)\end{vmatrix},\quad d_n=\begin{vmatrix}(\phi_0,\phi_0) & (\phi_0,\phi_1) & \cdots & (\phi_0,\phi_n)\\ (\phi_1,\phi_0) & (\phi_1,\phi_1) & \cdots & (\phi_1,\phi_n)\\ \cdots & & & \\ (\phi_n,\phi_0) & (\phi_n,\phi_1) & \cdots & (\phi_n,\phi_n)\end{vmatrix},$$
$$d_0(z)=\phi_0(z)\ ,\quad d_{-1}=d_0=1\ ,\quad (\phi_i,\phi_j)\equiv\int_{-\infty}^{\infty}\phi_i(z)\phi_j(z)P_y(z)dz\ . \tag{11}$$
and $\phi_i(z)$ is given in the orthonormal expansion expression of $P_y(z)=\sum_{i=0}^{\infty}\langle\phi_i(z)\rangle\phi_i(z)$. Especially after employing the probability expression of statistical Hermite series type as the above $P_y(y)$, we can consequently obtain the following expression (i.e., $\phi_i(z)$ corresponds to an Hermite polynomial of the ith order):
$$P_z(z)=\frac{1}{\sqrt{2\pi}\sigma_y}e^{-\frac{(z-\mu_y)^2}{2\sigma_y^2}}\sum_{r=0}^{\infty}\frac{B_r}{\sqrt{r!}}H_r\left(\frac{z-\mu_y}{\sigma_y}\right)\sum_{n=0}^{\infty}C_n\sum_{j=0}^{\infty}\frac{\lambda_{nj}}{\sqrt{j!}}H_j\left(\frac{z-\mu_y}{\sigma_y}\right) \tag{12}$$
with
$$C_n\equiv\sum_{j=0}^{n}\frac{\lambda_{nj}}{\sqrt{j!}}\left\langle H_j\left(\frac{z-\mu_y}{\sigma_y}\right)\right\rangle\ ,\quad (\phi_i,\phi_j)=\delta_{ij}+\sum_{r=0}^{\infty}B_r\frac{\sqrt{i!j!r!}}{(s-i)!(s-j)!(s-r)!}\ . \tag{13}$$

General framework of expressing output noise distribution

If we pay our special attention to the fact that the sound intensity always fluctuates only over non-negative region $[0,\infty)$, the output probability density function of sound insulation system can be first derived in a general form of a statistical Laguerre expansion series as follows [1] : $$P_z(z)=\frac{1}{\Gamma(m)s^m}e^{-z/s}z^{m-1}\{1+\sum_{n=3}^{\infty}D_nL_n^{(m-1)}(z/s)\} \tag{14}$$
with
$$m=\frac{\langle z\rangle^2}{\langle(z-\langle z\rangle)^2\rangle}\ ,\quad s=\frac{\langle(z-\langle z\rangle)^2\rangle}{\langle z\rangle}\ ,\quad D_n=\frac{\Gamma(m)n!}{\Gamma(m+n)}\langle L_n^{(m-1)}(z/s)\rangle\ . \tag{15}$$
The above parameters m, s and C_n $(n=1,2,\cdots)$ can be determined by using several of lower and higher order moments $\langle z^p\rangle$ $(p=1,2,\cdots,n)$ of output energy fluctuation z and the acoustic transmission characteristics a_ℓ after improvement of sound insulation system, as follows :
$$\langle z^p\rangle=\sum_{i_1,i_2,\cdots,i_p=1}^{N}a_{i_1}a_{i_2}\cdots a_{i_p}\langle z_{i_1}z_{i_2}\cdots z_{i_p}\rangle \tag{16}$$

Estimation of acoustic transmission characteristics

In two special cases when the sound insulation system is constructed by use of single- or double-walls, the acoustic transmission characteristics a_ℓ of each wall are explicitly expressed based on the usual and modified S.E.A. methods [2,3] as follows :

(Single-wall)
$$a_\ell=\frac{\begin{vmatrix}\eta_2+\eta_{21}+\eta_{23} & \eta_{12}\\ -\eta_{23} & \eta_{13}\end{vmatrix}}{\begin{vmatrix}\eta_2+\eta_{21}+\eta_{23} & -\eta_{32}\\ -\eta_{23} & \eta_3+\eta_{31}+\eta_{32}\end{vmatrix}}\cdot\frac{V_{in}}{V_{out}} \tag{17}$$

(Double-wall)

$$a_\ell = \frac{\begin{vmatrix} \eta_{2t} & -\eta_{32} & 0 & \eta_{12} \\ -\eta_{23} & \eta_{3t} & -\eta_{43} & \eta_{13} \\ 0 & -\eta_{34} & \eta_{4t} & 0 \\ 0 & -\eta_{35} & -\eta_{45} & \eta_{15} \end{vmatrix}}{\begin{vmatrix} \eta_{2t} & -\eta_{32} & 0 & 0 \\ -\eta_{23} & \eta_{3t} & -\eta_{43} & -\eta_{53} \\ 0 & -\eta_{34} & \eta_{4t} & -\eta_{54} \\ 0 & -\eta_{35} & -\eta_{45} & \eta_{5t} \end{vmatrix}} \cdot \frac{V_{in}}{V_{out}}$$

with

$$\eta_{2t}=\eta_2+\eta_{21}+\eta_{23}\ ,\quad \eta_{3t}=\eta_3+\eta_{31}+\eta_{32}+\eta_{34}+\eta_{35}\ ,\quad \eta_{4t}=\eta_4+\eta_{43}+\eta_{45}\ ,\quad \eta_{5t}=\eta_5+\eta_{51}+\eta_{53}+\eta_{54}\ . \qquad (18)$$

In the above equations, η_i is the internal loss factor, η_{ij} is the coupling loss factor. Moreover, V_{in} and V_{out} denote the volumes of transmission room and reception room respectively.

EXPERIMENTAL CONSIDERATION

We use an aluminium double-wall, or an aluminium single-wall (thickness : 1.2mm, surface-mass:3.22kg/m^2,loss factor:9.33 10^{-3},double-wall air gap :50mm), as two sound insulation systems. A one-third octave band noise of white noise is used as an input source. The used frequency range is from 250Hz to 1000Hz.We estimate the values of a_ℓ (ℓ=1,2,3,4,5,6,7) for single- and double-walls based on the modified and usual S.E.A. methods. The estimated values of a_ℓ for each case are shown in Table 1.

Fig.1 shows a comparison between theoretically evaluated curves (calculated by Eq.(6)) and experimental sample points of cumulative distribution for the transmitted sound level fluctuation in a typical case when the sound insulation system is changed from a single-wall to a double-wall.

Table 1 Estimated values of a_ℓ

1/3 octave band center frequency f_c (Hz)	Single-wall	Double-wall
250	4.546×10^{-2}	4.546×10^{-2}
315	3.058×10^{-2}	3.957×10^{-2}
400	1.855×10^{-2}	3.239×10^{-2}
500	1.119×10^{-2}	2.404×10^{-2}
630	5.712×10^{-3}	1.617×10^{-2}
800	2.862×10^{-3}	1.090×10^{-2}
1000	1.609×10^{-3}	6.895×10^{-3}

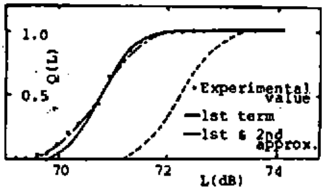

Fig.1 Comparison between theory and experiment for Q(L).

CONCLUDING REMARKS

In this paper, a new probabilistic evaluation method for the transmitted noise fluctuation of sound insulation system has been theoretically proposed by use of the usual and modified S.E.A. methods. An agreement between the theoretical and experimental probability expressions has been effectively confirmed. We would like to express our cordial thanks to S. Yamaguchi, K. Hatakeyama, Y. Ohira and A.Nanba for their helpful assistance.

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