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THE EFFECT OF TIME DELAY VARIATION INDUCED BY WORKPIECE EXCITATION ON REGENERATIVE CHATTER IN MILLING

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This paper describes the validation of the effect of time delay variation induced by workpiece excitation on regenerative chatter in end milling process. Regenerative chatter is one of the self-excited vibration phenomena caused by time delay. The various approaches have been proposed to prevent or to avoid the regenerative chatter occurrence. Among the countermeasures against regenerative chatter, the spindle speed variation is one of the effective techniques to suppress regenerative chatter. The spindle speed variation technique can suppress regenerative chatter by creating time varying delay and disturbing the regenerative effect. However, there are actually some constraint conditions in the speed variation amplitude and frequency in order to minimize the impact on the spindle life and reliability. In this work, instead of the spindle speed variation, the suppression effect on regenerative chatter is studied by exciting a workpiece in two orthogonal directions to change the relative velocity between a cutting edge and a workpiece at the cutting point. The excitation amplitude and frequency to improve the critical axial depth of cut at the onset of regenerative chatter is proved by conducting the stability analysis.

Keywords: machine tool vibration, self-excited vibration, regenerative chatter, variable time delay, vibration suppression

1. Introduction

Regenerative chatter in machine process causes a poor surface finish of a workpiece, a tool wear and a machine tool damage. Regenerative chatter is the self-excited vibration caused by regenerative effect (hereinafter, referred to as “chatter”). It is a very important issue for high-precision and high-efficiency machining to suppress chatter. The various approaches have been proposed to suppress chatter[1]. One of the passive damping techniques is the application of the tuned mass damper[1-4]. Using the tuned mass damper can increase the critical axial depth of cut at the onset of chatter. However, the tuned mass damper mounting position has to be selected to avoid tool pass and the tuning parameters of the damper has to be determined depending on the vibration mode related to chatter. On the other hand, the spindle speed variation is another technique to suppress chatter [1,5-7]. The spindle speed variation technique can suppress chatter by varying time delay and disturbing the regenerative effect. The parameters of the spindle speed variation can be changed easily according to the vibration mode related to chatter. However, there are actually some constraint conditions in the speed variation amplitude and frequency in order to minimize the impact on the spindle life and reliability.

In this work, instead of the spindle speed variation, the suppression effect on chatter was studied by exciting a workpiece in two orthogonal directions to change the relative velocity between a cutting edge and a workpiece at the cutting point. The excitation force to achieve the relative velocity variation is defined and the equation of motion with a variable time delay for the present model is derived. The excitation amplitude and frequency to improve the critical axial depth of cut at the onset of chatter is proved by conducting the stability analysis for this system using the semi-discretization method. Furthermore, the effect on chatter in the high frequency not possible with the conventional spindle speed variation is discussed.

2. Analysis

2.1 Analytical model

Figure 1 shows a workpiece model attached to a vibration stage capable of being excited in x-and y- directions individually. The workpiece can be excited in tangential direction by synchronous control of a tool revolution and each excitation direction of the vibration stage. Accordingly, it is possible to change the time delay by variating the relative velocity in tangential direction between the cutting edge and the workpiece in the cutting point. For simplicity, the present work supposes that the workpiece is modelled as a lumped mass system with one single degree of freedom in the cross feed direction (y-direction) as shown in Figure 2. The natural frequencies of the vibration stage in x- and y-directions are assumed to be much higher than the natural frequency of the workpiece. The end milling tool is assumed to be rigidly constrained and fed in the positive x- direction. In Figure 2, k_y and c_y are respectively the modal stiffness and the modal damping coefficient of the workpiece in y- direction. The tool rotates clockwise at an angular velocity of Ω. F is the cutting force acting on the workpiece. The direction of the cutting force varies with the tool revolution and depends on the instantaneous angular immersion ϕ_j of the j-th cutting edge of tool (hereafter, simply called the angular immersion) measured clockwise from y-direction.

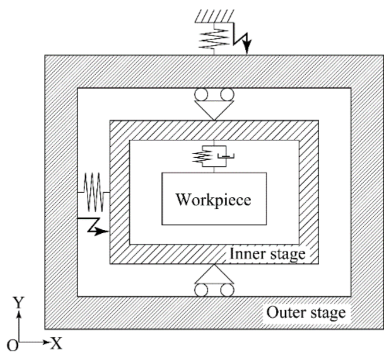


Figure 1. Workpiece model attached to vibration stage.

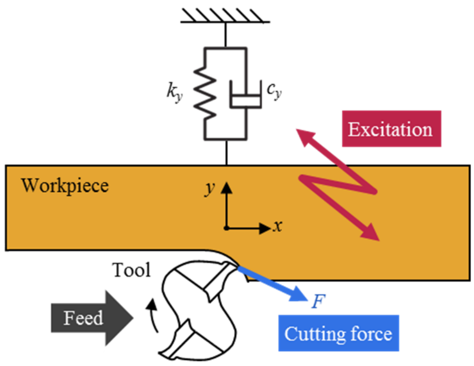


Figure 2. Workpiece model.

2.2 Derivation of variable time delay by workpiece excitation

This section describes the concept of chatter suppression method using the time delay variation by workpiece excitation. There have been many researches into the chatter suppression using spindle speed variation technique [5-7]. However, in case of the spindle speed variation technique, the spindle speed variation frequency is limited to up to one Hz at most because of the impact on the spindle life and the limitation of the motor torque. In the present work, we consider the implementation of the time delay fluctuation by variating the relative velocity in tangential direction between the cutting edge and the workpiece at the cutting point instead of spindle speed variation. The proposed method can excite the workpiece at high frequency. In this paper, the effect of the time delay variation at high frequency of several hundreds Hz or more on chatter is investigated. In order to variate the time delay by workpiece excitation, the excitation direction has to face the tangential direction of the cutting edge at the cutting point in synchronization with the tool rotation. To achieve that, it is required to provide the workpiece with the excitation displacement in *x*- and *y*- directions defined as the following equations when the end milling tool has two flutes in up-cut milling operation:

$$\begin{cases} x_e = A\cos\Omega t \sin\omega_e t \\ y_e = -A\sin\Omega t \sin\omega_e t \end{cases} . \tag{1}$$

where A is excitation amplitude and ω_e is excitation frequency. Figure 3 shows input waveforms to the workpiece in *x*-and *y*- directions expressed by the Eq. (1). In Fig. 3, τ_0 represents tooth passing period, which is expressed by the following equation:

$$\tau_0 = 2\pi / N_f \Omega = \pi / \Omega \quad (\because N_f = 2) . \tag{2}$$

where N_f is the number of teeth. As shown in Fig. 3, it is possible to fluctuate the relative velocity in tangential direction between the cutting edge and the workpiece at the cutting point by harmonizing a beat period with the tooth passing period. The excitation displacement in tangential direction is expressed by using the instantaneous angular immersion ϕ_j and the Eq. (1) :

$$\begin{aligned} u_{ej}(t) &= x_e(t)\cos\phi_j(t) - y_e(t)\sin\phi_j(t) \\ &= A\sin\omega_e t\left\{\cos\Omega t\cos\phi_j(t) + \sin\Omega t\sin\phi_j(t)\right\} \\ &= A\sin\omega_e t\left[\cos\left\{\Omega t - \phi_j(t)\right\}\right] \end{aligned} . \tag{3}$$

where the instantaneous angular immersion ϕ_j is defined as follows:

$$\phi_j(t) = \Omega\left\{t + (j-1)\tau_0\right\} \quad (j = 1, 2) . \tag{4}$$

When the number of teeth is two, the instantaneous angular immersion ϕ_j is expressed as follows:

$$\phi_j(t) = \begin{cases} \Omega t & (j = 1) \\ \Omega t + \pi & (j = 2) \end{cases} . \tag{5}$$

Hence, Eq.(3) is expressed as follows:

$$u_{ej}(t) = f(t, 2\tau_0) A\sin\omega_e t . \tag{6}$$

where the function *f* is defined as follows:

$$f(t, 2\tau_0) = \begin{cases} 1 & (\mathrm{mod}(t, 2\tau_0) < \tau_0) \\ -1 & (\mathrm{mod}(t, 2\tau_0) \geq \tau_0) \end{cases} . \tag{7}$$

When the workpiece is excited in tangential direction of a cutting edge in synchronization with the tool rotation, the instantaneous angular immersion is variated depending on the tool diameter. The variation of the instantaneous angular immersion $\Delta\phi(t)$ can be derived with Eq.(6) and the tool diameter d as shown in Fig.4 if the excitation displacement is small:

$$\begin{aligned} u_{ej}(t) &= \frac{d}{2}\Delta\phi(t) \\ \Delta\phi(t) &= \frac{2f(t, 2\tau_0)}{d} A\sin\omega_e t \end{aligned} . \tag{8}$$

When the workpiece is displaced in $+u_j$ direction as shown in Fig.4, the instantaneous angular immersion considering is reduced as expressed by the following equation:

$$\phi_j{}'(t) = \phi_j(t) - \Delta\phi(t) \ . \tag{9}$$

Since the instantaneous angular immersion during current cut is equal to that during previous cut, the following equation can be derived:

$$\phi_j{}'(t) = \phi_{j+1}{}'(t - \tau(t)) \ . \tag{10}$$

where $\tau(t)$ represents the time delay variation. Substituting Eq.(4) and Eq.(9) into Eq.(10) gives

$$\tau(t) = \frac{1}{\Omega}\left\{\pi + \Delta\phi(t) - \Delta\phi(t - \tau(t))\right\} \ . \tag{11}$$

Assuming that the time delay variation τ(t) in the right side in Eq.(11) is varied minutely on the basis of the tooth passing period, τ(t) is approximated by the following equation.

$$\tau(t) \approx \tau_0 \ . \tag{12}$$

Substituting Eq.(2), Eq.(8), and Eq.(12) into Eq.(11), the approximated time delay variation by the workpiece excitation can be expressed as follows:

$$\begin{aligned} \tau(t) &\approx \tau_0\left[1 + \frac{2A}{\pi d}\left\{f(t,2\tau_0)\sin\omega_e t - f(t-\tau_0,2\tau_0)\sin\omega_e(t-\tau_0)\right\}\right] \\ &= \tau_0\left[1 + \frac{2Af(t,2\tau_0)}{\pi d}\left\{\sin\omega_e t + \sin\omega_e(t-\tau_0)\right\}\right] \\ &= \tau_0\left[1 + \left\{\frac{4A}{\pi d}\cos\left(\frac{\omega_e\tau_0}{2}\right)\right\} f(t,2\tau_0)\sin\omega_e\left(t - \frac{\tau_0}{2}\right)\right] \end{aligned} \ . \tag{13}$$

Next, the non-dimensional excitation frequency is defined. From Eq.(13), the amplitude of the time delay variation depends on the excitation amplitude A and the excitation frequency ω_e. Since the excitation frequency and the tooth passing period are included in the argument of a cosine function, the amplitude of the time delay variation is zero depending on the product of the excitation frequency and the tooth passing period. Hence,

$$\cos\left(\frac{\omega_e\tau_0}{2}\right) = 0 \ \Rightarrow \ \frac{\omega_e\tau_0}{2} = \frac{\omega_e}{2\Omega}\pi = (m + \frac{1}{2})\pi \quad (m = 0,1,\cdots,\infty) \ . \tag{14}$$

In contrast, the condition to maximize the amplitude of the time delay variation can be expressed by

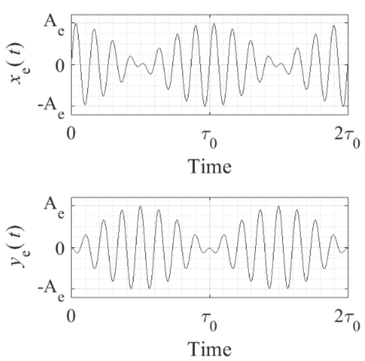


Figure 3. Input waveforms to workpiece in x- and y- directions.

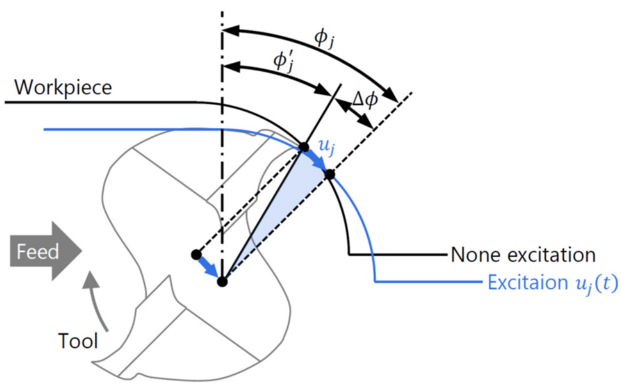


Figure 4. Relationship between excitation displacement and instantaneous angular immersion.

the following equation:

$$\cos\left(\frac{\omega_e \tau_0}{2}\right) = 1 \Rightarrow \frac{\omega_e \tau_0}{2} = \frac{\omega_e}{2\Omega}\pi = m\pi \quad (m = 0,1,\cdots,\infty) \;. \tag{15}$$

Therefore, in this paper the following non-dimensional excitation frequency is defined.

$$R_f = \frac{\omega_e}{2\Omega} \;. \tag{16}$$

From Eq.(13) and Eq.(15), it is predicted that the maximum chatter suppression effect can be obtained when the excitation frequency was equal to the integral multiple of a tooth passing frequency.

Next, the non-dimensional excitation amplitude is defined. The product of chatter period T_c and the angular velocity Ω represents a phase delay between the vibrations at successive tooth period. Accordingly, if the ratio of chatter period T_c to time delay $\tau_0 = \pi / \Omega$ is integer, the regenerative effect is minimum. The present work defines the ratio of the double amplitude of the instantaneous angular immersion variation $\Delta\phi(t)$ to the phase delay ΩT_c as the non-dimensional excitation amplitude R_a. Hence,

$$R_a = \frac{2\max[\Delta\phi(t)]}{\Omega T_c} = \frac{2A\omega_c}{\pi d\Omega} \approx \frac{2A\omega_y}{\pi d\Omega} \;. \tag{17}$$

where chatter frequency is approximated as the natural frequency of the workpiece in *y*- direction. Substituting Eq.(16) and Eq.(17) into Eq.(13), the approximated variable time delay by the workpiece excitation can be expressed as follows:

$$\tau(t) = \tau_0 + \frac{2\pi}{\omega_y} R_\mathrm{a} \cos\left(R_\mathrm{f}\pi\right) f(t, 2\tau_0) \sin\left(\frac{2\pi R_\mathrm{f}}{\tau_0} t - \pi\right) . \tag{18}$$

2.3 Equations of motion

In accordance with the description of Fig. 2, the equations of motion for the workpiece milling model subject to the time delay variation by workpiece excitation in tangential direction are obtained as follows:

$$\ddot{y} + 2\zeta_y \dot{y} + \omega_y^2 y = F_y / m \,. \tag{19}$$

where *m* is the modal mass of the workpiece and ζ_y, ω_y are respectively the damping ratio and the natural frequency of the workpiece in *y*- direction. The damping ratio and the natural frequency are defined as follows.

$$\zeta_y = c_y / 2\sqrt{mk_y} \,,\; \omega_y = \sqrt{k_y / m} \,. \tag{20}$$

F_y is the cutting force in *y*- direction acting on the workpiece and is expressed as follows:

$$F_y = \sum_{j=1}^{N_\mathrm{f}} F_{yj} = -F_{uj} \sin\phi_j(t) + F_{vj} \cos\phi_j(t) \,. \tag{21}$$

where F_{vj} and F_{uj} are respectively the cutting forces in the radial and tangential directions acting on the workpiece from the *j*-th cutting edge of the tool. In this analysis, the above cutting force ignore the effect on simultaneous cutting by multiple cutting edges because the end-milling tool has two cutting edges with a zero helix angle for simplicity. Therefore, the cutting force F_y is assumed as $F_y = F_{yj}$. F_{uj} and F_{vj} are proportional to the axial depth of cut *a* and the uncut chip thickness h_j in the direction of $\phi_j(t)$ and are defined as follows:

$$F_{uj} = K_u a h_j\left(\phi_j\left(t\right)\right),\ F_{vj} = K_v F_{uj}\,. \tag{22}$$

where the cutting coefficients K_u and K_v are constant. The uncut chip thickness $h_j(\phi_j(t))$ is expressed by the following equation.

$$h_j\left(\phi_j\left(t\right)\right) = \left[f\sin\phi_j\left(t\right) - \left\{y(t) - y(t-\tau(t))\right\}\cos\phi_j(t)\right]g\left(\phi_j\left(t\right)\right). \tag{23}$$

where f is the feed rate per a tooth and the function $g\left(\phi_j\left(t\right)\right)$ is introduced in order to evaluate whether the j-th cutting edge is engaged with the workpiece or not and is defined as follows:

$$g\left(\phi_j\left(t\right)\right) = \begin{cases} 1,\ (\phi_{\text{st}} < \phi_j(t) < \phi_{\text{ex}}) \\ 0,\ (\phi_j(t) < \phi_{\text{st}}, \phi_j(t) > \phi_{\text{ex}}) \end{cases}. \tag{24}$$

where ϕ_{st} is the start angle and ϕ_{ex} is the exit angle. In the present analysis, for up cut milling process is discussed. Therefore, $\phi_{\text{st}} = 0$ and $\phi_{\text{ex}} = \cos^{-1}(1 - r_d / r)$, where r is the radius of a tool and r_d is the radial depth of cut. Their angles define the angles at which the cutting edge enter and leave the workpiece respectively. The first term in Eq. (23) is the ideal chip thickness when the tool and spindle system is not vibrating and the other term is the dynamic fluctuation component between the vibrations at successive tooth period. Since the ideal chip thickness doesn't contribute to the stability of the system, the resultant cutting force can be derived by ignoring the term related to this ideal chip thickness as follows:

$$F_y = a\left\{-K_u\sin\phi_j(t) + K_v\cos\phi_j(t)\right\}\cos\phi_j(t)\left\{-y(t) + y(t-\tau(t))\right\}. \tag{25}$$

3. Numerical analysis

To verify the effect of the chatter stability lobe on the workpiece excitation and the predicted excitation frequency to obtain the maximum chatter suppression effect, the stability analysis for the workpiece milling model with time delay variation induced by workpiece excitation is performed.

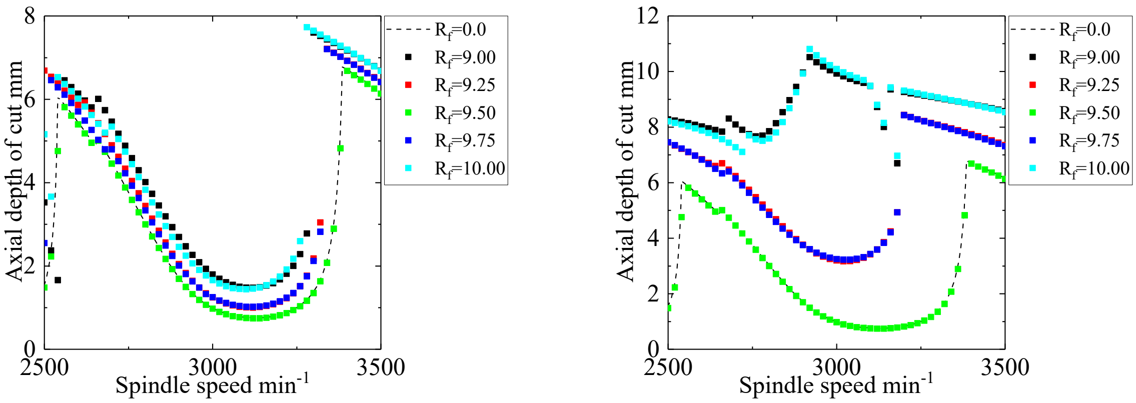
3.1 Chatter stability lobe with and without workpiece excitation

The workpiece milling model with time delay variation induced by workpiece excitation is the delayed oscillator with time periodic coefficients and time periodic delay. In the present study, the stability analysis for this delayed system is performed according to the first-order semi-discretization method proposed by Insperger et al.[8]. In the semi-discretization method, the stability properties is determined according to Floquet theory. The system has to be purely periodic to apply the Floquet theory of the delayed system with time periodic coefficients and time periodic delay. Since the time delay expressed by Eq.(18) is periodic at both the mean spindle speed period $2\tau_0$ and the period of time delay variable τ_0 / R_{f}, we assume that the ratio of $2\tau_0$ and τ_0 / R_{f} is a rational number, that is, $T = p(2\tau_0) = q(\tau_0 / R_{\text{f}})$ with T being the principal period of the system and p and q being relative primes. Thus, Floquet theory of the delayed oscillator with time periodic coefficients and time periodic delay can be applied. The critical depth of cut at onset of chatter is determined when the system is asymptotically stable.

First, the chatter stability lobe with and without workpiece excitation is discussed. Table 1 shows the calculation parameters used in the analysis. The modal parameters were determined by curve fitting of the frequency response function of the workpiece obtained from the hammering test of the workpiece[9]. Figure 5(a) and 5(b) show the chatter stability lobe with and without workpiece excitation at spindle speeds ranging from 2500 min^{-1} to 3500 min^{-1} obtained by the stability analysis. The abscissa is the spindle speed and the ordinate is the critical axial depth of cut at the onset of chatter. The dashed line represents the chatter stability lobe without exciattion. Each colored square represents the chatter stability lobe with excitation. The color scale represents the difference of non-dimenssion exctation frequency R_f. It was confrimed the spindle speed of 3100 min^{-1} corresponding to a local minimum of the chatter stability lobe obtained by the calculations gives close agreement with the previous results[9]. K in Table 1 represents the number of discrete steps over the principal period T in the analysis using the first order semi-discretization method. When R_a is less than 10%, the value of K is 1000 because the time delay variation is small. From Fig.5, when the excitation frequency is equal to the integral multiple of a tooth passing frequency as indicated in Eq.(15), the whole stability lobe is the most elevated. Meanwhile, when the excitation frequency was equal to (m+0.5) times of a tooth passing frequency as indicated in Eq.(14), the stability lobe is not changed. In addition, the larger the excitation amplitude is, the larger an increase in the critical depth of cut is.

Table 1: Calculation parameters used in analysis

M	1.73 kg	$\omega_y / 2\pi$	500 Hz	K_t	1.05×10^9 N/m^2	r	4.0 mm
K	400, 1000	ζ_y	0.006	K_r	0.0906	r_d	2.5 mm

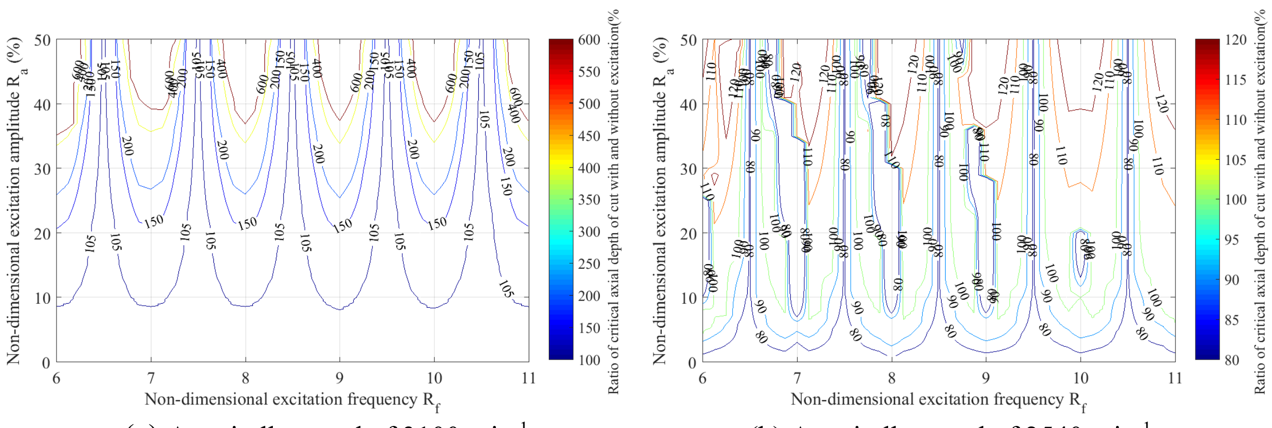


(a) R_a=25% (b) R_a=50%

Figure 5. Chatter stability lobe with and without excitation.

3.2 Effect of excitation frequency and excitation amplitude on chatter

Next, the effect of non-dimensional excitation frequency R_f and non-dimensional excitation amplitde R_a on the critical depth of cut at the spindle speed of the local minimum and local maximum of the chatter stability lobe without workpiece excitation is discussed. Figure 6(a) and 6(b) show the relationships between excitation frequency and excitation amplitude at spindle speed which are located in one of the local minima(3100 min^{-1}) and the local maxima(2540 min^{-1}) of the chatter stabity lobe. The color scale represents the critical depth of cut ratio with and without excitation. A larger value of this ratio indicates that the critical depth of cut is increased by excitation. From



(a) At spindle speed of 3100 min^{-1} (b) At spindle speed of 2540 min^{-1}

Figure 6. Relationship between excitation frequency and excitation amplitude.

Fig.6(a), non-dimensional excitation frequency R_f closer to an integer indicates more stable even though non-dimension excitation amplitude R_a is small. Meanwhile, from Fig.6(b), it can be seen that the critical depth of cut is decreased in small amplitude of R_a and R_f even though R_f is close to an integer. However, the crtical depth of cut is increased in a larege amplutude of R_f even though the amplitude of R_a is small. At present, we are conducting the experimental verification of the effect of time delay variation by workpiece excitation on chatter and will explore the effect on tool wear and machined surface quality.

4. Conclusions

In the present paper, we investigated analytically the effect of time delay variation induced by workpiece excitation on regenerative chatter in end milling process. The results are summarized as follows.

The optimum excitation frequency to effectively mitigate chatter was predicted by formulating the excitation condition to maximize the time delay fluctuation range. The stability analysis with the semi-discretization method revealed that an increase in the critical depth of cut at onset of chatter was maximized when the excitation frequency was equal to the integral multiple of a tooth passing frequency. Meanwhile, the critical depth of cut was not changed when the excitation frequency was equal to (m+0.5) times of a tooth passing frequency, where m is integer number. Furthermore, the stability analysis showed that the larger the excitation amplitude was, the larger an increase in the critical depth of cut was.

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