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ON THE COUPLING OF ACOUSTIC ENERGY BETWEEN A TRANSDUCER AND GLASS BEADS

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1. ABSTRACT

A novel approach for explaining the coupling of sound from transducers into granular materials is suggested. It is assumed that energy cannot be transferred through the point-contacts of the transducer and the grains. A thin, equivalent layer of air, henceforth labelled "the coupling layer", is proposed. The thickness of this layer is of the order of magnitude of the average grain size. Calculation of the coefficient of transmission using this assumption shows that the coupling is effectively a second order pole low-pass filter. An experiment to check on the validity of the model is carried out in glass beads of tightly controlled sizes. The results of the model and experiment agree within the experimental error, pointing to a decrease in the pole frequency as the bead size increases.

2. BACKGROUND

In many practical instances in seismics and acoustics one is considered with the coupling of acoustic energy between a transducer and a medium. In seismic work, the mandatory procedure is to wet the soil around the geophone to enhance energy transfer. It is clear that the moist layer is only a small fraction of a wavelength. Similarly, most underwater transducers are externally moulded in rubber (or polyurethane) to increase the amount of energy flowing between them and water. Again the rubber layer thickness is much smaller than a wavelength. Moreover, the water-tightness of a transducer can be achieved using techniques other than external rubber coating. The common denominator among all the above procedures of the type is that a thin layer relative to a wavelength is applied to the interface between the transducer and the medium to improve the acoustic energy coupling.

The most prevalent method of formulating the coupling layer in underwater acoustics (1) or in architectural acoustics (2), is to include it as a specific coupling stratum in a multi-layered system including source and receiver. The same mechanism may also apply to sound propagation through the soil (3). Nevertheless, most previous work (4-8), has not considered the coupling between transducer and soil, attempting instead to formulate an explanation of the sound propagation in terms of wave types. One recent work (9) interpreted coupling in terms of experimental errors but considered only seismic frequencies.

This paper suggests that a coupling layer should be incorporated into models of the acoustic energy propagation from or to a transducer in a soil. Also, an experiment in the sonic kHz range was carried out with glass beads as the granular medium.

3. THEORY

Assume a layer structure as in Fig. 1. Layer one is the transducer, layer two is the "coupling" layer and layer three is the soil, or any porous medium. As the grains of the porous medium touch the transducer only atpoints, this interface is not really capable of an energy transfer (although it can transfer a mechanical field). The basic assumption of this work is, therefore, that between the transducer and the otherwise continuous porous medium there is an equivalent layer of air the thickness of which is of the order of magnitude of the grain size in the medium.

As indicated in Fig.1, once can associate a density, and sound speed with each layer. Assume that the acoustic ray is incident on the z=0 interface between the transducer and air at an angle θ_1. This ray is refracted at an angle θ_2 into the coupling air layer. Travelling through the air layer the thickness of which is 1, the ray is refracted again at the air-soil interface to emerge at an angle θ_3. In a practical situation, a transducer would be directional requiring an integration on the θ_1 incident angle. One may obtain θ_2 and θ_3 using Snell's law. The calculation shows that the propagation in the transducers is almost one dimensional, and that the maximum value of θ_3 is about 12°. Although θ_2 goes up to 90°, one can nevertheless assume a one dimensional energy problem. Since the air gap between the transducer and the grains is much smaller than the diameter of the transducer only a negligible amount of energy will be lost at high θ_2.

Introducing the sub-index m (m = 1, 2, 3) as the layer index, and n (n = i, t, r for incident, transmitted and reflected) as the wave-type index, one can write the pressure of the wave moving in the direction $\vec{d}$ as:

$$p_m = A_{n,m} \exp\left[j\left(\omega t - d/\lambda_m\right)\right] \tag{1}$$

$$= A_{n,m} \exp\left[j\left(\omega t - \frac{z\cos\theta_m}{\lambda_m} - \frac{x\sin\theta_m}{\lambda_m}\right)\right]$$

$$u_m = \frac{p_m}{\rho_m c_m} \tag{2}$$

where ρ_m, c_m are the density and the speed of sound in the layer m, and d is resolved as shown in Fig 1, and $Z_m = \rho_m c_m$ are the impedances.

One can use the conservation laws for the pressure and particle velocity (eqs. (2), (3)) at the z=0, z=l interfaces:

$$\left.\begin{aligned} p_{i,1} + p_{r,1} &= p_{t,2} + p_{r,2} \\ u_{i,1} + u_{r,1} &= u_{t,2} + u_{r,2} \end{aligned}\right\} z = 0 \text{ interface} \tag{3}$$

$$\left.\begin{aligned} p_{t,2} + p_{r,2} &= p_{t,3} \\ u_{t,2} + u_{r,2} &= u_{t,3} \end{aligned}\right\} z = l \text{ interface} \tag{4}$$

Substituting the pressures (eq. (1)) and the particle velocities (eq. (2)) into the conservation laws (eq. (3) and eq. (4)), and using the definition of the required attenuation of energy between layers 1 and 3:

$$a_t = \left(\frac{A_3}{A_1}\right)\frac{\rho_1 c_1}{\rho_3 c_3} \tag{5}$$

One obtains after some algebra:

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$$a_t = \frac{4Z_1Z_3\cos\theta_2}{D} \tag{6-1}$$

$$D = (Z_1\cos\theta_2\cos\theta_3 + Z_3\cos\theta_2)^2\cos^2(k_2l\ \cos\theta_2) + (Z_2\cos\theta_3 + Z_1Z_3\cos^2\theta_2/Z_2)^2\sin^2(k_2l\cos\theta_2) \tag{6-2}$$

where $k2 = \frac{2\pi}{\lambda 2}$ is the wave number for layer 2.

This expression is true both for the transmitter-coupling-beads path (TB) and for the beads-coupling-receiver (BR) path. It was also assumed the $\cos\theta_1 = 1$ since both transducers are one dimensional to a first approximation.

Both eqs. (6) and (8) can be written as:

$$a_t = \frac{\alpha}{\beta\cos\theta_2(k_2l\cos\theta_2) + \gamma\sin^2(k_2l\cos\theta_2)} \tag{7}$$

where α, β and γ constants are independent of frequency and bead size.

Assuming $k_2l \ll 1$, equivalent to the coupling layer being very small compared to a wavelength, one obtains:

$$a_t = \frac{\alpha}{\beta + \gamma(k_2l)^2} \tag{8}$$

For the sake of simplicity $\theta_2 = 0$ was chosen. As one can see, at very low frequencies ($k_2 \to 0$)

$$a_L = \frac{\alpha}{\beta} \tag{9}$$

and the attenuation is constant. While at very high frequencies the constant term in the denominator is small and therefore:

$$a_H = \frac{\alpha}{\gamma\left(\frac{2\pi l}{c2}\right)^2 f^2} \tag{10}$$

In this case the attenuation behaves as a second order pole at $f = 0$. The functional behaviour of a_t is illustrated in Fig.2. One can obviously see that there is a transition frequency, f_t, at which the transmission coefficient changes its frequency response. This transition frequency f_t is given by the conditions $a_L = a_H$, therefore:

$$f_t = \frac{c2}{2\pi l}\left(\frac{\beta}{\gamma}\right)^{\frac{1}{2}} \tag{11}$$

and will depend upon l, and hence upon the bead size.

The values of α, β, γ can be expressed using the impedances for the TB case:

$$\alpha = 4 Z_1{}^2 Z_3 \quad (12\text{-}1)$$

$$\beta = 4 Z_1 Z^2{}_2 \quad (12\text{-}2)$$

$$\gamma = Z_1{}^3 \quad (12\text{-}3)$$

Using the values of Tables 1 and 2, one obtains the values of Z_1, Z_2, Z_3 and f_t for the TB and BR cases (see Table 2) and 630 μ beads. One should note that the frequency dependence of the attenuation comes through k_2, but may depend also on the values of the impedances, if variation of the sound speeds with frequency is allowed. In this work it was assumed that any frequency dependence is represented by k_2 only, and that all sound speeds are frequency independent.

Substance	Impedance (mks rayls)	Density (gm^3)	Sound speed (m/s^{-1})
aluminium	1.53×10^7	2.78	5500
glass beads	1.27×10^5	1.72	74
air	4.15×10^2	1.21×10^{-3}	343 (at 20°)

Table 1. Densities, sound speeds and impedances, for aluminium, glass beads and air

Case	Interface	z_1 (ray/s)	z_2 (ray/s)	z_3 (ray/s)	f_t (H_z)
TB	xmitter - beads	5.3×10^7	4.15×10^2	3.35×10^5	74
BR	beads - receiver	3.35×10^5	4.15×10^2	15.3×10^7	3380

Table 2. Predicted transition frequencies for Cases TB and BR

Table 2 indicates that f_t for the TB case is very low (less than 100Hz) and, probably, it would be very difficult to verify experimentally. The value predicted of f_t for the BR case depends on the glass bead size: the larger the bead, the lower f_t. The calculated value of a_L yields no attenuation up to f_{tTB}. Hence in a situation when glass beads are located between the faces of identical transmitter and receiver the qualitative behaviour of a_t is predicted to be as follows:

a. negligible attenuation up to f_{tTB}.

b. 6 dB/oct. excess attenuation between f_{tTB} and f_{tBR}.

c. 12 dB/oct excess attenuation above f_{tBR}.

The results of the numerical calculation carried out for several glass bead sizes appear in Fig. 3 for an assumed sound speed of 74 m/sec in the glass beads. The sound speed in the glass beads was not measured, since a long range propagation experiment was not carried out. The assumed value of 74 m/sec for the sound speed was borrowed from a long range (~ 100 wavelengths) experiment (not reported here) using silver sand with the same grain size distribution as the glass beads.

4. EXPERIMENT

An experiment was carried out to test the predictions of the model in section 2. The experimental set-up appears in Fig. 4., while the electronic system for the data acquisition is illustrated in Fig. 5.

Two identical transducers of the sandwich type were supplied by British Aerospace. Both are rejected units, which is demonstrated by spurious resonances attesting to defects in manufacturing, and therefore the useful frequency range for observation was 8-16 kHz only. The transducers resonate at 21 kHz and have an aluminium transmitting (receiving) head of 30 mm diameter.

A stand-bench was constructed to locate both transducers on the same axis facing each other. The holders of the transducers were acoustically isolated from each other using a succession of aluminium and nylon discs providing an excellent mismatch of −110 dB with respect to a case in which both transducers had face contact. The apparatus includes a micrometric movement used to fix the transducer faces at a predetermined distance of 4 mm. This distance was chosen to provide at least 8-10 glass beads in the transmitter-receiver path. The 30 mm head diameter provides an area large enough to integrate over the various possible paths between the two transducers. The transducers were encased and electrically screened by the Engineering Mechanics machine shop.

The heart of the data acquisition system was an FFT analyser, Ono-Sokki CF–920. Besides allowing for a dual channel acquisition of both transmission and reception data it generated the signal for the experiment. The transmitting branch included a B&K 4440 gating system which trimmed the continuous signal into single, manually activated bursts of 4 msecs. The pulses thus obtained were amplified using a B&K 2706 amplifier. The output was then stepped up using an in-house transformer providing a more convenient match between the amplifier and the transmitter. The receiving branch included of two amplifiers: A B&K 2635 charge amplifier was connected to the receiver and its output fed into a B&K 5612 spectrum shaper was used as a band-pass filter on the filter loop of the line B&K 2607 amplifier, thus preventing seismic noise on one hand and the transducer resonance on the other from limiting the dynamic range of the measurement. The outputs of the power amplifier in the transmitting branch and the line amplifier in the receiving branch were connected to the FFT analyser which was used only as an ADC and mass storage device.

The following experiments were carried out, each using 16 bursts:

a. Transmitter and receiver at intimate contact. Labelled llr.

b. 4 mm air gap between the transducers. This experiment was used to test whether cross-correlation produced the speed of sound in air.

c. 4 mm gap between the transducers, but filled consecutively with various distributions of glass beads. The average grain sizes of the distributions were: 25μm, 70μm, 160μm, 340μm, 630μm, 1125μm.

The data analysis produced the transfer function between transmission and reception in each experiment. To eliminate the system response and the effect of the individual transducer, transfer functions were taken of the transfer functions relative to that for the transmitter and receiver in contact. The only spectral range of interest was the one found with almost zero phase difference, although a small linear phase dependence was allowed due to the 5612 filter in the reception path. This range was found to be consistent in all the experiments and determined to be approximately 9-16.5 kHz. At last, the slope of the coupling attenuation in dB/octave in the 9.155 kHz-16.48 kHz band is calculated using a linear fit.

5. RESULTS AND DISCUSSION

Figure 6 is a typical example of the transfer functions (each in two frames - amplitude & phase). The slope of the transfer is plotted in a solid line over the dotted experimental one. Also, the slope results, together with the model predictions, are given in Table 3. As seen, the phase drops from 1 radian at 9.155 kHz to about 0 radians at 16.48 kHz. The 2π radians jumps are artefacts of the calculation only, while the slight drop in the phase is a result of the filtering in the receiving branch.

Glass bead size (microns)	Calculated slope* (dB octave)	Measured slope (dB octave)
25	-4.99	-5.51 ± 1.5
70	-5.16	-3.89 ± 1.5
160	-5.65	-6.29 ± 1.5
340	-6.98	-8.42 ± 1.5
630	-8.47	-11.34 ± 1.5
1125	-9.43	-11.29 ± 1.5

*sound speed in glass beads assumed to be 74 m/sec.

Table 3. Experimental and calculated slopes for the coupling attenuation in various glass bead sizes

The proposed theoretical model is very simplistic. The propagation between two- transducers considered to involve two couplings only. No account is given to any propagation within the glass beads themselves. At a sound speed of 74 m/sec, the distance between the two transducers at 9.155-16 kHz is 4.62 mm, is barely a wavelength or two. There is not enough gap, therefore, between the transducers to allow significant propagation effects through the glass beads.

The experimental set-up chosen immediately eliminates the low frequency region, where no relevant effects can be measured due to a low signal to noise ratio. The comparison of the theoretical and experimental results points at the trend of an increasing attenuation vs frequency slope with increasing bead size.

The scope of the experiment was much hindered by the narrow usable bandwidth of the transducers. Better transducers would have yielded not only a wider frequency attenuation constant, but would have enabled a transmitted - received signal cross-correlation measurement without many prominant side lobes. One could have then retrieved the sound speed in the coupling layer from the cross-correlation measurement, possibly with an investigation of the dispersion of sound speed vs bead size. The assumption that α,β,γ (see eqs. (12)) are frequency independent may have also been removed, resulting in a study of the propagation in the coupling layer.

6. ACKNOWLEDGEMENTS

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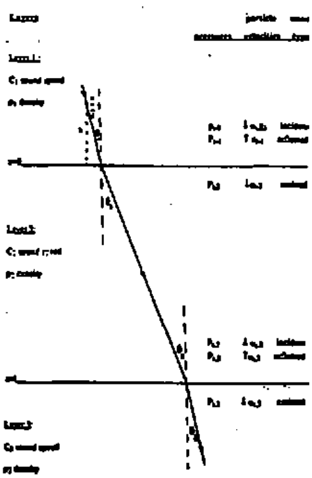
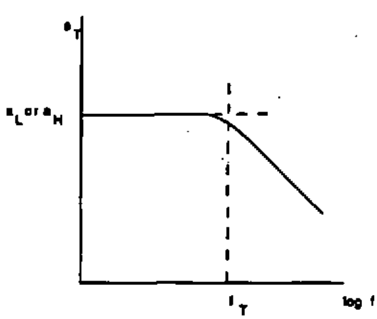
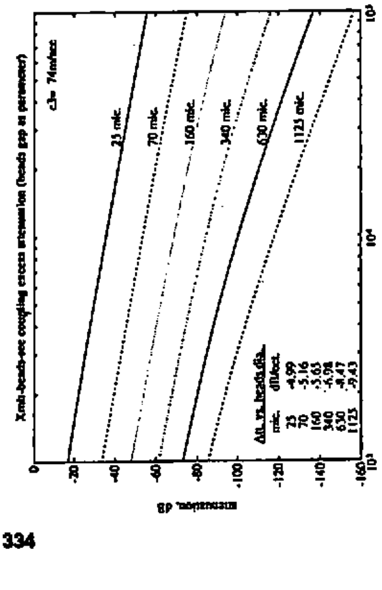
8. FIGURES



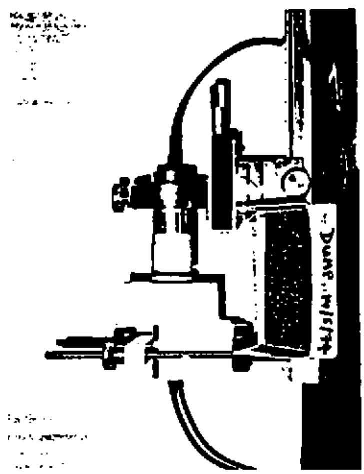
1. Schematic description of the 3-layered coupling interface.



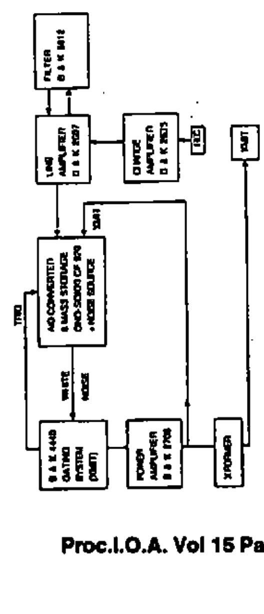
2. The functional shape of the attenuation versus frequency.



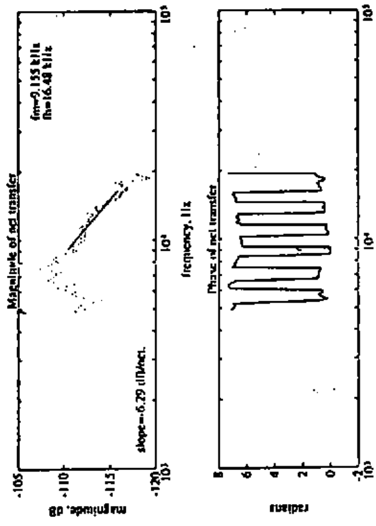
3. Calculated attenuation for various glass bead sizes. and sound speed of 74 m/sec.



4. The mechanical set-up.



5. The electronic measurement system.



6. Transfer function - amplitude and phase-for coupling of 160μm beads.