IMPACT RESPONSE OF A NONLINEAR VISCOELASTIC AUXETIC DOUBLY CURVED SHALLOW SHELL

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Impact induced large amplitude (geometrically non-linear) vibrations of doubly curved shallow viscoelastic shells with rectangular base are investigated for the case when the shear operator is governed by the fractional derivative Kelvin-Voigt model in conjunction with the time-independent coefficient of volume extension-compression, what is verified by experimental data. It has been shown that such a model could describe the behaviour of so-called auxetic materials with negative Poisson's ratios. It is assumed that the shell is simply supported and partial differential equations are obtained in terms of shell's transverse displacement and Airy's stress function. The equations of motion are reduced to a set of infinite nonlinear ordinary differential equations of the second order in time and with cubic and quadratic nonlinearities in terms of the generalized displacements. Assuming that only two natural modes of vibrations dominate during the process of impact, the method of multiple time scales in conjunction with the expansion of the fractional derivative in terms of a small parameter has been utilized for solving nonlinear governing equations of motion.
Keywords: nonlinear vibrations, fractional derivative, auxetics, impact response

1. Introduction

A review of the literature devoted to dynamic behavior of curved panels and shells could be found in [1,2]. Nonlinear vibrations of doubly curved shallow shells induced by the low-velocity impact by an elastic sphere were investigated in [3,4]. The dynamic response of an initially flat viscoelastic membrane was considered in [5] utilyzing the fractional derivative Kelvin-Voigt model and time-independent Poisson's ratio what is in conflict with experimental data.

In spite of the fact that the impact theory is substantially developed, there is a limited number of papers devoted to the problem of impact over geometrically nonlinear viscoelastic shells, the review of which could be found in [3,4].

In the present paper, a new approach has been proposed for the analysis of the impact interactions of nonlinear viscoelastic doubly curved shallow shells with rectangular base under the low-velocity impact by an elastic sphere. To describe the damping features of the shell, the shear operator is preassigned in terms of the fractional derivative Kelvin-Voigt model

$$\tilde{\mu} = \mu_0[1 + (\tau_\sigma^\mu)^\gamma D_{0+}^\gamma], \qquad (1)$$

where μ_0 is the relaxed shear modulus, τ_σ^μ is the retardation time, $\gamma\,(0 < \gamma \le 1)$ is the fractional parameter, D_{0+}^γ is the Riemann-Liouville fractional derivative [6,7]

$$D_{0+}^\gamma x(t) = \frac{d}{dt}\int_0^t \frac{x(t')dt'}{\Gamma(1-\gamma)(t-t')^\gamma}, \qquad (2)$$

$\Gamma(1-\gamma)$ is the Gamma function, and $x(t)$ is an arbitrary function.

The bulk operator is assumed to be time-independent $\tilde{K} = K_0 = \text{const}$, i.e,

$$\tilde{E} = \frac{9K_0\tilde{\mu}}{3K_0 + \tilde{\mu}}. \tag{3}$$

It has been shown that such a model could describe the behaviour of so-called auxetic materials with negative Poisson's ratios [8].

Let us write Poisson's operator $\tilde{\nu}$ in the form [8]

$$\tilde{\nu} = -1 + \frac{E_0}{2\mu_0} \text{э}_\gamma^* (t_\sigma^\gamma), \tag{4}$$

where $\text{э}_\gamma^* (t_\sigma^\gamma)$ is the Rabotnov dimensionless fractional operator defined as follows [6]

$$\text{э}_\gamma^* (t_\sigma^\gamma) = \frac{1}{1 + t_\sigma^\gamma D_{0+}^\gamma}. \tag{5}$$

It is assumed that the shell is simply supported and partial differential equations are obtained in terms of shell's transverse displacement and Airy's stress function. The equations of motion are reduced to a set of infinite nonlinear ordinary differential equations of the second order in time and with cubic and quadratic nonlinearities in terms of the generalized displacements. Assuming that only two natural modes of vibrations dominate during the process of impact and applying the method of multiple time scales, the set of recurrence equations of various orders is obtained.

2. Problem formulation and governing equations

Assume that an elastic or rigid sphere of mass M moves along the z-axis towards thin-walled doubly curved shell with thickness h, curvilinear lengths a and b, principle curvatures k_x and k_y and rectangular base, as shown in Fig. 1. Impact occurs at the moment $t = 0$ with the low velocity εV_0 at the point N with Cartesian coordinates x_0, y_0, where ε is a small dimensionless parameter.

According to the Donnell-Mushtari nonlinear shallow shell theory, the equations of motion could be obtained in terms of lateral deflection w and Airy's stress function ϕ [9]

$$\frac{\tilde{D}}{h}\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 \phi}{\partial x^2} + k_y\frac{\partial^2 \phi}{\partial x^2} + k_x\frac{\partial^2 \phi}{\partial y^2} - 2\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 \phi}{\partial x \partial y} + \frac{F}{h} - \rho\ddot{w}, \tag{6}$$

$$\frac{1}{\tilde{E}}\left(\frac{\partial^4 \phi}{\partial x^4} + 2\frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4}\right) = -\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - k_y\frac{\partial^2 w}{\partial x^2} - k_x\frac{\partial^2 w}{\partial y^2}, \tag{7}$$

where $\tilde{D} = \dfrac{\tilde{E}h^3}{12(1-\tilde{\nu}^2)}$ is the cylindrical rigidity operator, ρ is the density, $\tilde{E}$ and $\tilde{\nu}$ are Young's operator and Poisson's operator, respectively, t is time, $F = P(t)\delta(x - x_0)\delta(y - y_0)$ is the contact force, $P(t)$ is yet unknown function, δ is the Dirac delta function, x and y are Cartesian coordinates, overdots denote time-derivatives, $\phi(x, y)$ is the stress function which is the potential of the in-plane force resultants

$$N_x = h\frac{\partial^2 \phi}{\partial y^2}, \ N_y = h\frac{\partial^2 \phi}{\partial x^2}, \ N_{xy} = -h\frac{\partial^2 \phi}{\partial x \partial y}. \tag{8}$$

The equation of motion of the sphere is written as

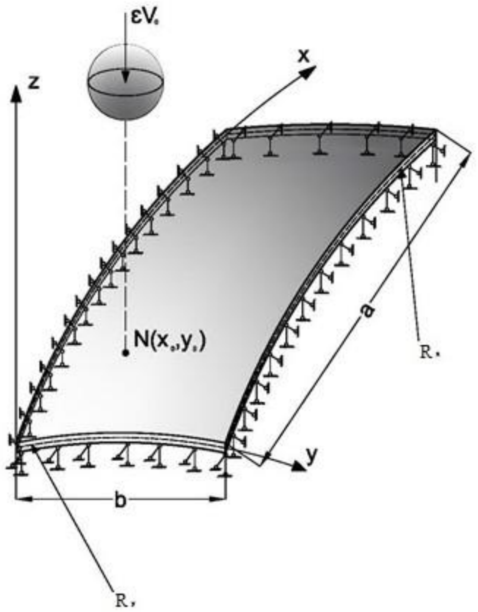

Figure 1: Geometry of a doubly curved shallow shell

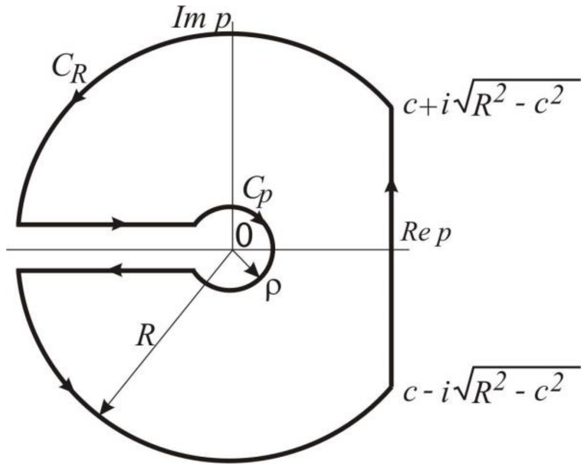

Figure 2: The contour of integration

$$M\ddot{z} = -P(t) \tag{9}$$

subjected to the initial conditions

$$z(0) = 0, \quad \dot{z}(0) = \varepsilon V_0, \tag{10}$$

where $z(t)$ is the displacement of the sphere, in so doing

$$z(t) = w(x_0, y_0, t). \tag{11}$$

Considering a simply supported shell with movable edges, the following conditions should be imposed at each edge: at $x = 0, a$

$$w = 0,\ \int_0^b N_{xy} dy = 0,\ N_x = 0,\ M_x = 0, \tag{12}$$

and at $y = 0, b$

$$w = 0,\ \int_0^a N_{xy} dx = 0,\ N_y = 0,\ M_y = 0, \tag{13}$$

where M_x and M_y are the moment resultants.

The suitable trial function that satisfies the geometric boundary conditions is

$$w(x, y, t) = \sum_{p=1}^{\tilde{p}} \sum_{q=1}^{\tilde{q}} \xi_{pq}(t) \sin\left(\frac{p\pi x}{a}\right) \sin\left(\frac{q\pi y}{b}\right), \tag{14}$$

where p and q are the number of half-waves in x and y directions, respectively, and $\xi_{pq}(t)$ are the generalized coordinates. Moreover, $\tilde{p}$ and $\tilde{q}$ are integers indicating the number of terms in the expansion.

Substituting (14) in (11) and using (9), we obtain

$$P(t) = -M \sum_{p=1}^{\tilde{p}} \sum_{q=1}^{\tilde{q}} \ddot{\xi}_{pq}(t) \sin\left(\frac{p\pi x_0}{a}\right) \sin\left(\frac{q\pi y_0}{b}\right). \tag{15}$$

In order to find the solution of the set of Eqs. (6) and (7), it is necessary first to obtain the solution of Eq. (7). For this purpose, let us substitute (14) in the right-hand side of Eq. (7) and seek the solution of the equation obtained in the form

$$\phi(x,y,t)=\sum_{m=1}^{\tilde{m}}\sum_{n=1}^{\tilde{n}}A_{mn}(t)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \tag{16}$$

where $A_{mn}(t)$ are yet unknown functions.

Substituting (14) and (16) in Eq. (7) and using the orthogonality conditions of sines within the segments $0\le x\le a$ and $0\le y\le b$, we have

$$A_{mn}(t)=\frac{\tilde{E}}{\pi^2}K_{mn}\xi_{mn}(t)+\frac{4\tilde{E}}{a^3b^3}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^{-2}\sum_k\sum_l\sum_p\sum_q B_{pqklmn}\xi_{pq}(t)\xi_{kl}(t), \tag{17}$$

where coefficients K_{mn} and B_{pqklmn} are presented in [3,4].

Substituting then (14)-(17) in Eq. (6), using the orthogonality condition of sines within the segments $0\le x\le a$ and $0\le y\le b$ and multiplying equation by operator $\tilde{J}=\tilde{E}^{-1}$, we obtain an infinite set of coupled nonlinear ordinary differential equations for defining the generalized coordinates

$$\begin{aligned}&\ddot{\xi}_{mn}(t)\tilde{J}+\tilde{\Omega}^2_{mn}\xi_{mn}(t)+\frac{8\pi^2}{a^3b^3\rho}\sum_p\sum_q\sum_k\sum_l B_{pqklmn}\left(K_{kl}-\frac{1}{2}K_{mn}\right)\xi_{pq}(t)\xi_{kl}(t)\\&+\frac{32\pi^4}{a^6b^6\rho}\sum_r\sum_s\sum_i\sum_j\sum_k\sum_l\sum_p\sum_q B_{rsijmn}B_{pqklij}\xi_{rs}(t)\xi_{pq}(t)\xi_{kl}(t)\\&+\tilde{J}\,\frac{4M}{ab\rho h}\sin\left(\frac{m\pi x_0}{a}\right)\sin\left(\frac{n\pi y_0}{b}\right)\sum_p\sum_q\ddot{\xi}_{pq}(t)\sin\left(\frac{p\pi x_0}{a}\right)\sin\left(\frac{q\pi y_0}{b}\right)=0,\end{aligned} \tag{18}$$

where $\tilde{\Omega}_{mn}$ is the viscoelastic operator corresponding to the natural frequency of the mn th mode of the shell vibration defined as

$$\tilde{\Omega}^2_{mn}=\frac{1}{\rho}\left[\frac{\pi^4h^2}{12(1-\tilde{\nu}^2)}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2+K_{mn}\right]. \tag{19}$$

Let us find the operator $(1+\tilde{\nu}^2)^{-1}$, which with due account for (4) could be presented in the following form:

$$\frac{1}{1-\tilde{\nu}^2}=\frac{1}{2}\left[\frac{1}{1+\tilde{\nu}}+\frac{1}{1-\tilde{\nu}}\right]=\frac{1}{2}\left[\frac{3+\nu_0}{1+\nu_0}+\frac{1}{1+\nu_0}t_\sigma^\gamma D^\gamma+\frac{1+\nu_0}{2(1-\nu_0)}\,\text{э}^*_\gamma\,(At_\sigma^\gamma)\right]. \tag{20}$$

Now let us calculate the operator inverse to the operator (3)

$$\tilde{J}=\frac{1}{9K_0}\left[1+\frac{3K_0}{\mu_0}\,\text{э}^*_\gamma\,(\tau_\sigma^\mu)^\gamma\right]. \tag{21}$$

Then we could rewrite operator $\tilde{\Omega}_{mn}$ considering (20), that is

$$\tilde{\Omega}^2_{mn}=\Omega^2_{0mn}+d_{1mn}t_\sigma^\gamma D^\gamma+d_{2mn}\,\text{э}^*_\gamma\,(At_\sigma^\gamma), \tag{22}$$

where

$$\Omega^2_{0mn}=\frac{\pi^4}{\rho}\frac{h^2}{24}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2\frac{3+\nu_0}{1+\nu_0}+\frac{K_{mn}}{\rho},\qquad A=\frac{2}{1-\nu_0},\qquad \nu_0=\frac{\lambda_0}{2(\lambda_0+\mu_0)},$$

$$d_{1mn} = \frac{\pi^4}{\rho}\frac{h^2}{24}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2\frac{1}{1+\nu_0};\ d_{2mn} = \frac{\pi^4}{\rho}\frac{h^2}{48}\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2\frac{1+\nu_0}{1-\nu_0}.$$

The last term in each equation from (18) describes the influence of the coupled impact interaction of the target with the impactor of the mass M applied at the point with the coordinates x_0, y_0.

It is known [10] that during nonstationary excitation of thin bodies not all possible modes of vibration would be excited. Moreover, the modes which are strongly coupled by any of the so-called internal resonance conditions are initiated and dominate in the process of vibration, in so doing the types of modes to be excited are dependent on the character of the external excitation.

Thus, in order to study the additional nonlinear phenomenon induced by the coupled impact interaction due to equation (18), we suppose that only two natural modes of vibrations are excited during the process of impact, namely, $\Omega_{\alpha\beta}$ and $\Omega_{\gamma\delta}$, in so doing each type of impact subjection should be considered separately.

3. Method of solution

Let us consider first the case of strong action of the impact force upon a target. Then the set of Eqs (18) is reduced to the following two nonlinear differential equations:

$$\tilde{J}(p_{11}\ddot{\xi}_{\alpha\beta} + p_{12}\ddot{\xi}_{\gamma\delta}) + \tilde{\Omega}^2_{\alpha\beta}\xi_{\alpha\beta} + p_{13}\xi^2_{\alpha\beta} + p_{14}\xi^2_{\gamma\delta} + p_{15}\xi_{\alpha\beta}\xi_{\gamma\delta} + p_{16}\xi^3_{\alpha\beta} + p_{17}\xi_{\alpha\beta}\xi^2_{\gamma\delta} = 0, \quad (23)$$

$$\tilde{J}(p_{21}\ddot{\xi}_{\alpha\beta} + p_{22}\ddot{\xi}_{\gamma\delta}) + \tilde{\Omega}^2_{\gamma\delta}\xi_{\gamma\delta} + p_{23}\xi^2_{\gamma\delta} + p_{24}\xi^2_{\alpha\beta} + p_{25}\xi_{\alpha\beta}\xi_{\gamma\delta} + p_{26}\xi^3_{\gamma\delta} + p_{27}\xi^2_{\alpha\beta}\xi_{\gamma\delta} = 0, \quad (24)$$

where

$$p_{11} = 1+\frac{4M}{\rho hab}s_1^2,\quad p_{22} = 1+\frac{4M}{\rho hab}s_2^2,\quad p_{12} = p_{21} = \frac{4M}{\rho hab}s_1 s_2,\quad s_1 = \sin\left(\frac{\alpha\pi x_0}{a}\right)\sin\left(\frac{\beta\pi y_0}{b}\right),$$

$$s_2 = \sin\left(\frac{\gamma\pi x_0}{a}\right)\sin\left(\frac{\delta\pi y_0}{b}\right),\ p_{13} = \frac{8\pi^2}{a^3b^3\rho}B_{\alpha\beta\alpha\beta\alpha\beta}\frac{1}{2}K_{\alpha\beta},\quad p_{14} = \frac{8\pi^2}{a^3b^3\rho}B_{\gamma\delta\gamma\delta\alpha\beta}\left(K_{\gamma\delta}-\frac{1}{2}K_{\alpha\beta}\right),$$

$$p_{15} = \frac{8\pi^2}{a^3b^3\rho}\left[B_{\gamma\delta\alpha\beta\alpha\beta}\frac{1}{2}K_{\alpha\beta} + B_{\alpha\beta\gamma\delta\alpha\beta}\left(K_{\gamma\delta}-\frac{1}{2}K_{\alpha\beta}\right)\right],\quad p_{23} = \frac{8\pi^2}{a^3b^3\rho}B_{\alpha\beta\alpha\beta\gamma\delta}\left(K_{\alpha\beta}-\frac{1}{2}K_{\gamma\delta}\right),$$

$$p_{24} = \frac{8\pi^2}{a^3b^3\rho}B_{\gamma\delta\gamma\delta\gamma\delta}\frac{1}{2}K_{\gamma\delta},\qquad p_{25} = \frac{8\pi^2}{a^3b^3\rho}\left[B_{\alpha\beta\gamma\delta\gamma\delta}\frac{1}{2}K_{\gamma\delta} + B_{\gamma\delta\alpha\beta\gamma\delta}\left(K_{\alpha\beta}-\frac{1}{2}K_{\gamma\delta}\right)\right],$$

$$p_{16} = \frac{32\pi^2}{a^3b^3\rho}\sum_i\sum_j B_{\alpha\beta ij\alpha\beta}B_{\alpha\beta\alpha\beta ij},\ p_{17} = \frac{32\pi^2}{a^3b^3\rho}\sum_i\sum_j\left(B_{\alpha\beta ij\alpha\beta}B_{\gamma\delta\gamma\delta ij} + B_{\gamma\delta ij\alpha\beta}B_{\alpha\beta\gamma\delta ij} + B_{\gamma\delta ij\alpha\beta}B_{\gamma\delta\alpha\beta ij}\right),$$

$$p_{26} = \frac{32\pi^2}{a^3b^3\rho}\sum_i\sum_j B_{\gamma\delta ij\gamma\delta}B_{\gamma\delta\gamma\delta ij},\ p_{27} = \frac{32\pi^2}{a^3b^3\rho}\sum_i\sum_j\left(B_{\alpha\beta ij\gamma\delta}B_{\gamma\delta\gamma\delta ij} + B_{\gamma\delta ij\gamma\delta}B_{\alpha\beta\gamma\delta ij} + B_{\gamma\delta ij\gamma\delta}B_{\gamma\delta\alpha\beta ij}\right).$$

In order to solve a set of two nonlinear equations (23) и (24), we apply the method of multiple time scales [11] via the following expansions:

$$\xi_{ij}(t) = \varepsilon X^1_{ij}(T_0,T_1,T_2) + \varepsilon^2 X^2_{ij}(T_0,T_1,T_2) + \varepsilon^3 X^3_{ij}(T_0,T_1,T_2), \quad (25)$$

where $ij = \alpha\beta$ or $\gamma\delta$, $T_n = \varepsilon^n t$ are new independent variables, among them: $T_0 = t$ is a fast scale characterizing motions with the natural frequencies, and $T_1 = \varepsilon t$ and $T_2 = \varepsilon t^2$ are slow scales characterizing the modulation of the amplitudes and phases of the modes with nonlinearity.

Recall that the first and the second time derivatives are defined, respectively, as follows

$$\frac{d}{dt} = D_0 + \varepsilon D_0 + \varepsilon^2 D_2 + ..., \tag{26}$$

$$\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 \left(D_1^2 + 2D_0 D_2 \right), \tag{27}$$

where $D_i^n = \partial^n / \partial T_i^n$ ($n = 1, 2,\ i = 0,1$).

The fractional derivative is interpreted as the fractional power of the differential operator [12]

$$D_+^\gamma = \left(\frac{d}{dt} \right)^\gamma = D_0^\gamma + \varepsilon\gamma D_0^{\gamma-1} D_1 + ..., \tag{28}$$

Let us note that [7]

$$\left(\frac{d}{dt} \right)^\gamma \varphi = D_+^\gamma \varphi = \frac{d}{dt} \int_{-\infty}^{t} \frac{\varphi(t')dt'}{(t-t')^\gamma \Gamma(1-\gamma)}. \tag{29}$$

But in the present case the process of vibrations starts at the time $t = 0$. Thus we should adopt the fractional derivative in the following form:

$$D_{0+}^\gamma \varphi = \frac{d}{dt} \int_0^t \frac{\varphi(t')dt'}{(t-t')^\gamma \Gamma(1-\gamma)}. \tag{30}$$

Fractional derivatives (29) and (30) act differently on the function $e^{\lambda t}$, which is utilized in further treatment, i.e.

$$D_+^\gamma e^{\lambda t} = \lambda^\gamma e^{\lambda t}, \tag{31}$$

and

$$D_{0+}^\gamma e^{\lambda t} = \lambda^\gamma e^{\lambda t} + \frac{\sin \pi x}{\pi} \int_0^\infty \frac{u^\gamma}{u+\lambda} e^{-ut} du \,. \tag{32}$$

However, as it has been shown in [12], if in the method of multiple time scales only the zero and first order approximations are considered, then the second term in (32) could be neglected.

Now let us expand the Rabotnov dimensionless fractional operator (5) in a Taylor series. As a result we have

$$\begin{aligned} \text{Э}_\gamma^* (\tau^\gamma) &= \frac{1}{1+\tau^\gamma D_{0+}^\gamma} = (1+\tau^\gamma D_{0+}^\gamma)^{-1} = \left[1 + \tau^\gamma (D_0^\gamma + \varepsilon\gamma D_0^{\gamma-1} D_1) \right]^{-1} \\ &= (1+\tau^\gamma D_0{}^\gamma)^{-1} - \varepsilon (1+\tau^\gamma D_0{}^\gamma)^{-2} \tau^\gamma \gamma D_0^{\gamma-1} D_1 + ... \end{aligned} \tag{33}$$

The same procedure that has been used above for calculating $D_{0+}^\gamma e^{\lambda t}$ can be utilized for calculating the expression

$$f(t) = \text{Э}_\gamma^* (\tau^\gamma) e^{\lambda t} = \frac{1}{1+\tau^\gamma D_{0+}^\gamma} e^{\lambda t}. \tag{34}$$

For this purpose let us apply the Laplace transform to Eq. (34)

$$\overline{f(t)} = \overline{\text{Э}_\gamma^* (\tau^\gamma) e^{\lambda t}} = \frac{1}{[1+(p\tau)^\gamma]} \frac{1}{(p-\lambda)}, \tag{35}$$

where an overbar denotes the Laplace transform, and p is the Laplace variable.

Passing from the image to the original in Eq. (35), we have

$$f(t) = \text{Э}_\gamma^* (\tau^\gamma) e^{\lambda t} = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{1}{[1+(p\tau)^\gamma]} \frac{1}{(p-\lambda)} e^{pt} dp = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \overline{f}(p) e^{pt} dp \,. \tag{36}$$

Using the contour of integration shown in Fig. 2 we could reduce Eq. (36) to the following form:

$$f(t)=\frac{1}{2\pi i}\int_0^\infty[\bar{f}(se^{-i\pi})-\bar{f}(se^{i\pi})]e^{-st}ds+\sum_R[\bar{f}(p_R)e^{p_R t}]. \tag{37}$$

Considering (35) in (37), we find

$$f(t)=\frac{1}{1+(\lambda\tau)^\gamma}e^{\lambda t}-\frac{\sin\pi\gamma}{\pi}\int_0^\infty\frac{e^{-ut}du}{[(u\tau)^\gamma+(u\tau)^{-\gamma}+2\cos\pi\gamma](u+\lambda)}, \tag{38}$$

where

$$\frac{1}{1+(\lambda\tau)^\gamma}e^{\lambda t}=\frac{1}{1+\tau^\gamma D_+^\gamma}e^{\lambda t}. \tag{39}$$

The second term in (38) could be neglected, if in the method of multiple time scales jnly the zero and first order approximation are considered.

Substituting relationships (25)-(33) in (23) and (24), after equating the coefficients at like powers of ε to zero, we are led to a set of recurrence equations to various orders:

to order ε

$$p_{11}D_0^2\tilde{J}_0X_1^1+p_{12}D_0^2\tilde{J}_0X_2^1+\tilde{\Omega}_1^2X_1^1=0, \tag{40}$$

$$p_{21}D_0^2\tilde{J}_0X_1^1+p_{22}D_0^2\tilde{J}_0X_2^1+\tilde{\Omega}_2^2X_2^1=0; \tag{41}$$

to order ε^2

$$\begin{aligned}&p_{11}D_0^2\tilde{J}_0X_1^2+p_{12}D_0^2\tilde{J}_0X_2^2+\tilde{\Omega}_1^2X_1^2=-2p_{11}D_0D_1\tilde{J}_0X_1^1-2p_{12}D_0D_1\tilde{J}_0X_2^1\\&+\tilde{I}_{11}\gamma D_0^{\gamma-1}D_1(p_{11}D_0^2X_1^1+p_{12}D_0^2X_2^1)+\tilde{I}_{12}\gamma D_0^{\gamma-1}D_1X_1^1-p_{13}(X_1^1)^2-p_{14}(X_2^1)^2-p_{15}X_1^1X_2^1,\end{aligned} \tag{42}$$

$$\begin{aligned}&p_{21}D_0^2\tilde{J}_0X_1^2+p_{22}D_0^2\tilde{J}_0X_2^2+\tilde{\Omega}_2^2X_2^2=-2p_{21}D_0D_1\tilde{J}_0X_1^1-2p_{22}D_0D_1\tilde{J}_0X_2^1\\&+\tilde{I}_{11}\gamma D_0^{\gamma-1}D_1(p_{21}D_0^2X_1^1+p_{22}D_0^2X_2^1)+\tilde{I}_{22}\gamma D_0^{\gamma-1}D_1X_2^1-p_{23}(X_1^1)^2-p_{24}(X_2^1)^2-p_{25}X_1^1X_2^1,\end{aligned} \tag{43}$$

to order ε^3

$$\begin{aligned}&p_{11}D_0^2\tilde{J}_0X_1^3+p_{12}D_0^2\tilde{J}_0X_2^3+\tilde{\Omega}_1^2X_1^3=-2p_{11}D_0D_1\tilde{J}_0X_1^2-2p_{12}D_0D_1\tilde{J}_0X_2^2\\&-p_{11}\left(D_1^2+2D_0D_2\right)\tilde{J}_0X_1^1-p_{12}\left(D_1^2+2D_0D_2\right)\tilde{J}_0X_2^1+p_{11}\tilde{I}_{11}\gamma D_0^{\gamma-1}D_1(D_0^2X_1^2+2D_0D_1X_1^1)\\&+p_{12}\tilde{I}_{11}\gamma D_0^{\gamma-1}D_1(D_0^2X_2^2+2D_0D_1X_2^1)+\tilde{I}_{12}\gamma D_0^{\gamma-1}D_1X_1^2-2p_{13}X_1^1X_1^2-2p_{14}X_2^1X_2^2\\&-p_{15}\left(X_1^1X_2^2+X_1^2X_2^1\right)-p_{16}\left(X_1^1\right)^3-p_{17}X_1^1\left(X_2^1\right)^2,\end{aligned} \tag{44}$$

$$\begin{aligned}&p_{21}D_0^2\tilde{J}_0X_1^3+p_{22}D_0^2\tilde{J}_0X_2^3+\tilde{\Omega}_2^2X_2^3=-2p_{21}D_0D_1\tilde{J}_0X_1^2-2p_{22}D_0D_1\tilde{J}_0X_2^2\\&-p_{21}\left(D_1^2+2D_0D_2\right)\tilde{J}_0X_1^1-p_{22}\left(D_1^2+2D_0D_2\right)\tilde{J}_0X_2^1+p_{21}\tilde{I}_{11}\gamma D_0^{\gamma-1}D_1(D_0^2X_1^2+2D_0D_1X_1^1)\\&+p_{22}\tilde{I}_{11}\gamma D_0^{\gamma-1}D_1(D_0^2X_2^2+2D_0D_1X_2^1)+\tilde{I}_{22}\gamma D_0^{\gamma-1}D_1X_2^2-2p_{23}X_2^1X_2^2-2p_{24}X_1^1X_1^2\\&-p_{25}\left(X_1^1X_2^2+X_1^2X_2^1\right)-p_{26}\left(X_2^1\right)^3-p_{27}\left(X_1^1\right)^2X_2^1,\end{aligned} \tag{45}$$

where for simplicity is it denoted $X_1^1=X_{\alpha\beta}^1$, $X_2^1=X_{\gamma\delta}^1$, $X_1^2=X_{\alpha\beta}^2$, $X_2^2=X_{\gamma\delta}^2$, $\tilde{\Omega}_1=\tilde{\Omega}_{\alpha\beta}$, and $\tilde{\Omega}_2=\tilde{\Omega}_{\gamma\delta}$ and

$$\tilde{J}_0=\frac{1}{9K_0}\left[1+\frac{3K_0}{\mu_0}(1+\tau_\sigma^{\mu\gamma}D_0^\gamma)^{-1}\right];\quad \tilde{I}_{11}=\frac{1}{3\mu_0}(1+\tau_\sigma^{\mu\gamma}D_0^\gamma)^{-2}\tau_\sigma^{\mu\gamma};$$

$$\tilde{I}_{12}=-d_{11}t_\sigma^\gamma+d_{21}(1+At_\sigma^\gamma D_0^\gamma)^{-2}At_\sigma^\gamma.$$

4. Conclusion

Large amplitude (geometrically non-linear) vibrations of doubly curved shallow viscoelastic shells with rectangular base were investigated for the case when the shear operator is governed by the fractional derivative Kelvin-Voigt model in conjunction with the time-independent coefficient of volume extension-compression, what is verified by experimental data. It has been shown that such a model could describe the behaviour of so-called auxetic materials with negative Poisson's ratios. Due to their unique properties auxetics have potentially important applications in the nearest future. Auxetics can be advantageously used in the development of hydrophones and other sensors; for fibre reinforcement in composites; shock and sound absorbers; fasteners and rivets; air filters; for thermal protection in aerospace; for the manufacture of defence protective clothing [13].

It has been assumed that the shell is simply supported and partial differential equations have been obtained in terms of shell's transverse displacement and Airy's stress function, in so doing the local bearing of the shell and impactor's materials was neglected with respect to the shell deflection in the contact region. The governing set of nonlinear differential equations has been obtained using the method of multiple time scales.

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