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FLEXURAL VIBRATION TRANSMISSION AT JUNCTIONS

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1. Introduction

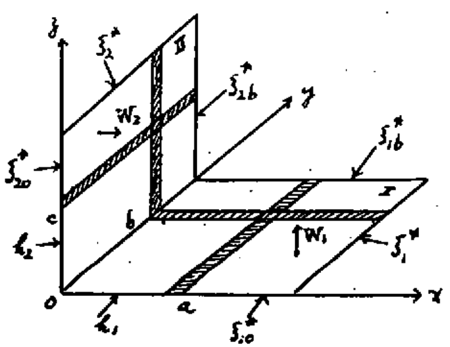
Statistical energy analysis (SEA) methods are often used in the calculation of power flows between coupled structural elements and the vibration levels of each element of a system. The method gives only the relationships between average quantities and the results are usually unreliable in the low frequency range where model densities are low; a deterministic solution is therefore preferred although the calculations are more complicated if not intractable.

This paper describes an approximate method used to calculate the bending vibrations of a combination of rectangular thin plates with elastically supported and damped non-coupled edges subjected to point sinusoidal excitations. The method has clear applications in the prediction and control of machine induced vibrations in buildings or in any case where low frequency vibration sources are coupled to a system of beams and plates. The method is similar to that used for calculating the bending vibrations of a single rectangular plate where the displacement amplitude function is expressed as a linear combination of coordinate functions and is extended to the case of combination of plates. The displacement amplitude function vector of the global system of plates is also expressed as a linear combination of coordinate function vectors which satisfy all the boundary conditions of the global system.

Two examples, an L-combination and a series of T-combinations of rectangular thin plates, are discussed and the input and transfer mobilities derived. The frequency response of combinations of concrete plates is calculated in the frequency range below 500 Hz, and the effect of material damping and edge losses is investigated.

2. L-combination of two rectangular plates

2.1 Differential equations



The governing differential equations of bending vibration of the L-combination of rectangular thin plates (as shown in Fig.1) can be written as

Fig.1.

$$\begin{pmatrix} L_1 & 0 \\ 0 & L_2 \end{pmatrix} \begin{pmatrix} w_1(x,y,t) \\ w_2(y,z,t) \end{pmatrix} = \begin{pmatrix} q_1(x,y,t) \\ q_2(y,z,t) \end{pmatrix} \tag{1}$$

where $L_i \equiv D_i^* \nabla_i^4 - \rho_i h_i \frac{\partial^2}{\partial t^2}$, $i = 1, 2$; $\nabla_1^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial y^2} + \frac{\partial^4}{\partial y^4}$,
$\nabla_2^4 = \frac{\partial^4}{\partial y^4} + 2\frac{\partial^2}{\partial y^2}\frac{\partial^2}{\partial z^2} + \frac{\partial^4}{\partial z^4}$.

ρ_i, h_i, $D_i^*(=D_i(1+j\eta))$ are respectively the density, thickness and complex bending stiffness of the ith plate. $\begin{pmatrix} w_1(x,y,t) \\ w_2(y,z,t) \end{pmatrix}$ and $\begin{pmatrix} q_1(x,y,t) \\ q_2(y,z,t) \end{pmatrix}$ are displacement function and pressure function vectors.
If the excitations are sinusoidal i.e.

$$\begin{pmatrix} q_1(x,y,t) \\ q_2(x,y,t) \end{pmatrix} = \begin{pmatrix} Q_1(x,y) \\ Q_2(y,z) \end{pmatrix} e^{j\omega t} \tag{2}$$

where ω is the radian frequency and $\begin{pmatrix} Q_1(x,y) \\ Q_2(y,z) \end{pmatrix}$ is the pressure amplitude function vector applied to the global system, then equation (1) reduces to

$$\begin{pmatrix} \mathcal{L}_1 & 0 \\ 0 & \mathcal{L}_2 \end{pmatrix} \begin{pmatrix} W_1(x,y) \\ W_2(y,z) \end{pmatrix} = \begin{pmatrix} Q_1(x,y) \\ Q_2(y,z) \end{pmatrix} \tag{3}$$

where $\mathcal{L}_i \equiv D_i^* \nabla_i^4 - \omega^2 \rho_i h_i$, $i = 1, 2$ and $\begin{pmatrix} W_1(x,y) \\ W_2(y,z) \end{pmatrix}$ is the bending displacement amplitude function vector.

2.2 Boundary conditions

The boundary conditions can be written as:

On y = o, z = o edge;

$$W_1(x,0) = 0$$

$$D_1^*\left(\frac{\partial^2 W_1}{\partial y^2} + \nu_1 \frac{\partial^2 W_1}{\partial x^2}\right)_{y=0} = \zeta_{10}^*\left(\frac{\partial W_1}{\partial y}\right)_{y=0}$$

On $y = b$, $z = 0$ edge;

$$W_1(x,b) = 0$$

$$D_1^*\left(\frac{\partial^2 W_1}{\partial y^2} + \nu_1 \frac{\partial^2 W_1}{\partial z^2}\right)_{y=b} = -\zeta_{1b}^*\left(\frac{\partial W_1}{\partial y}\right)_{y=b} \tag{4}$$

On $x = a$, $z = 0$ edge;

$$W_1(a,y) = 0$$

$$D_1^*\left(\frac{\partial^2 W_1}{\partial x^2} + \nu_1 \frac{\partial^2 W_1}{\partial y^2}\right)_{x=a} = -\zeta_1^*\left(\frac{\partial W_1}{\partial x}\right)_{x=a} \tag{5}$$

On $x=0,\ z=0$ edge;

$$\left.\begin{aligned}
&W_1(0,y)=0,\quad W_2(y,0)=0\\
&\left.\frac{\partial W_1(x,y)}{\partial x}\right|_{x=0}+\left.\frac{\partial W_2(y,z)}{\partial z}\right|_{z=0}=0\\
&D_1^*\left(\frac{\partial^2 W_1}{\partial x^2}+\nu_1\frac{\partial^2 W_1}{\partial y^2}\right)_{x=0}=D_2^*\left(\frac{\partial^2 W_2}{\partial z^2}+\nu_2\frac{\partial^2 W_2}{\partial y^2}\right)_{z=0}
\end{aligned}\right\}\qquad(6)$$

On $x=0,\ y=0$ edge;

$$W_2(0,z)=0,\quad D_2^*\left(\frac{\partial^2 W_2}{\partial y^2}+\nu_2\frac{\partial^2 W_2}{\partial z^2}\right)_{y=0}=\zeta_{2c}^*\left(\frac{\partial W_2}{\partial y}\right)_{y=0}$$

On $x=0,\ y=b$ edge;

$$W_2(b,z)=0,\quad D_2^*\left(\frac{\partial^2 W_2}{\partial y^2}+\nu_2\frac{\partial^2 W_2}{\partial z^2}\right)_{y=b}=-\zeta_{2b}^*\left(\frac{\partial W_2}{\partial y}\right)_{y=b}\qquad(7)$$

On $x=0,\ z=c$ edge;

$$\left.\begin{aligned}
&W_2(y,c)=0,\\
&D_2^*\left(\frac{\partial^2 W_2}{\partial z^2}+\nu_2\frac{\partial^2 W_2}{\partial y^2}\right)_{z=c}=-\zeta_2^*\left(\frac{\partial W_2}{\partial z}\right)_{z=c}
\end{aligned}\right\}\qquad(8)$$

$\nu_i,\ \zeta_i^*,\ \zeta_{io}^*,\ \zeta_{ib}^*$ are respectively the Poisson's ratio and the boundary complex rotational stiffnesses (as shown in Fig.1) of the ith plate, i = 1,2.

2.3 Coordinate function vectors

Let $K_{1x\ell}$, $K_{2z\ell}$, $\Phi_{1x\ell}(x)$, $\Phi_{2z\ell}(z)$ be the ℓth eigen wave numbers and eigen functions of the L-combination of beams of unit width perpendicular to the coupled edge (the shaded areas in Fig.1) which satisfy the boundary conditions corresponding to equations (5), (6) and (8).

Let K_{1ym}, K_{2ym}, $\Phi_{1ym}(y)$, $\Phi_{2ym}(y)$ be respectively the mth eigen wave numbers, eigen functions of the single beams of unit width parallel to the coupled edge, lying in Plate I and Plate II and satisfying the boundary conditions corresponding to equations (4) and (7). For simplicity, assume $\zeta_{1o}^*/D_1^*=\zeta_{2o}^*/D_2^*$ and $\zeta_{1b}^*/D_1^*=\zeta_{2b}^*/D_2^*$ then the two single beams have same eigen wave numbers and eigen functions.

$$K_{1ym}=K_{2ym}=K_{ym},\quad \Phi_{1ym}(y)=\Phi_{2ym}(y)=\Phi_{ym}(y),\quad m=1,\cdots,M.$$

The (ℓ, m,n) th coordinate function vector for the global system can be written as:

$$\begin{pmatrix}W_{1\ell m}(x,y)\\ W_{2n\ell}(y,z)\end{pmatrix}=\begin{pmatrix}\Phi_{1x\ell}(x)\,\Phi_{ym}(y)\\ \Phi_{yn}(y)\,\Phi_{2z\ell}(z)\end{pmatrix}\qquad(9)$$

satisfying the boundary conditions on the non-coupled edges. In order to satisfy the boundary conditions on the coupled edge, m = n, and the coordinate function vector (9) reduces to:

$$\begin{pmatrix} W_{1\ell m}(x,y) \\ W_{2m\ell}(y,z) \end{pmatrix} = \begin{pmatrix} \Phi_{1x\ell}(x)\,\Phi_{ym}(y) \\ \Phi_{ym}(y)\,\Phi_{2z\ell}(z) \end{pmatrix} \tag{10}$$

2.4 Approximate solution

The approximate solution to equation (3) can be written as:

$$\begin{pmatrix} W_1(x,y) \\ W_2(y,z) \end{pmatrix} = \sum_{\ell=1}^{L}\sum_{m=1}^{M} A_{\ell m} \begin{pmatrix} \Phi_{1x\ell}(x)\,\Phi_{ym}(y) \\ \Phi_{ym}(y)\,\Phi_{2z\ell}(z) \end{pmatrix} \tag{11}$$

Substituting (11) into (3) yields.

$$\begin{pmatrix} \mathcal{L}_1 & 0 \\ 0 & \mathcal{L}_2 \end{pmatrix} \sum_{\ell}\sum_{m} \begin{pmatrix} \Phi_{1x\ell}(x)\,\Phi_{ym}(y) \\ \Phi_{ym}(y)\,\Phi_{2z\ell}(z) \end{pmatrix} A_{\ell m} = \begin{pmatrix} Q_1(x,y) \\ Q_2(y,z) \end{pmatrix} + \begin{pmatrix} \varepsilon_1(x,y) \\ \varepsilon_2(y,z) \end{pmatrix} \tag{12}$$

where $\begin{pmatrix} \varepsilon_1(x,y) \\ \varepsilon_2(y,z) \end{pmatrix}$ is an error function vector.

Multiplying both sides of equation (12) by the operator vector $\bar{V}_{\ell' m'}$

$$\bar{V}_{\ell' m'} \equiv \left(\int_0^b \int_0^a \tilde{\Phi}_{1x\ell'}(x)\,\tilde{\Phi}_{ym'}(y)(\quad)\,dx\,dy,\; \int_0^c \int_0^b \tilde{\Phi}_{ym'}(y)\,\tilde{\Phi}_{2z\ell'}(z)(\quad)\,dy\,dz \right)$$

where $\tilde{\Phi}_{1x\ell'}, \tilde{\Phi}_{ym'}, \tilde{\Phi}_{2z\ell'}$ are respectively the complex conjugate functions of $\Phi_{1x\ell'}, \Phi_{ym'}, \Phi_{2z\ell'}$, and applying Galerkin's orthogonality condition for the global system $\bar{V}_{\ell' m'} \cdot \begin{pmatrix} \varepsilon_1(x,y) \\ \varepsilon_2(y,z) \end{pmatrix} = 0$, yields

$$\sum_{\ell=1}^{L}\sum_{m=1}^{M} T(\ell', m', \ell, m)\, A_{\ell m} = F_{\ell' m'} \tag{13}$$

where $T(\ell', m', \ell, m) = \bar{V}_{\ell' m'} \cdot \begin{pmatrix} \mathcal{L}_1 & 0 \\ 0 & \mathcal{L}_2 \end{pmatrix} \begin{pmatrix} \Phi_{1x\ell}(x)\,\Phi_{ym}(y) \\ \Phi_{ym}(y)\,\Phi_{2z\ell}(z) \end{pmatrix}$

$$= \int_0^b \int_0^a \tilde{\Phi}_{1x\ell'}\,\tilde{\Phi}_{ym'}\,\mathcal{L}_1\,\Phi_{1x\ell}\,\Phi_{ym}\,dx\,dy + \int_0^c \int_0^b \tilde{\Phi}_{ym'}\,\tilde{\Phi}_{2z\ell'}\,\mathcal{L}_2\,\Phi_{ym}\,\Phi_{2z\ell}\,dy\,dz\,; \tag{14}$$

$$F_{\ell' m'} = \bar{V}_{\ell' m'} \cdot \begin{pmatrix} Q_1(x,y) \\ Q_2(y,z) \end{pmatrix} = \int_0^b \int_0^a \tilde{\Phi}_{1x\ell'}\,\tilde{\Phi}_{ym'}\,Q_1\,dx\,dy + \int_0^c \int_0^b \tilde{\Phi}_{ym'}\,\tilde{\Phi}_{2z\ell'}\,Q_2\,dy\,dz\,; \tag{15}$$

$$\ell' = 1, \cdots, L\,;\quad m' = 1, \cdots, M.$$

Equation (13) can be written in matrix form

$$\begin{pmatrix} T(1,1,1,1) & \cdots & T(1,1,L,M) \\ \vdots & \ddots & \vdots \\ T(L,M,1,1) & \cdots & T(L,M,L,M) \end{pmatrix} \begin{pmatrix} A_{11} \\ \vdots \\ A_{LM} \end{pmatrix} = \begin{pmatrix} F_{11} \\ \vdots \\ F_{LM} \end{pmatrix}, \tag{16}$$

$\begin{pmatrix} A \\ \vdots \\ A_{LM} \end{pmatrix}$ and $\begin{pmatrix} F_{11} \\ \vdots \\ F_{LM} \end{pmatrix}$ being generalised coordinate and generalised force vectors.

$A_{\ell m}$'s can be solved from equation (16) and by substituting $A_{\ell m}$'s into equation (11) one gets the solution to equation (3). The complex eigen frequencies and eigen vectors of the global system can also be calculated from equation (16) letting the generalized force vector equal to zero.

Let (S) be the inverse matrix of the (T) matrix in equation (16) and $S(\ell,m,\ell',m')$ be the terms of (S) matrix, then equation (16) can be written as:

$$\begin{pmatrix} A_{11} \\ \vdots \\ A_{LM} \end{pmatrix} = \left(S \right) \begin{pmatrix} F_{11} \\ \vdots \\ F_{LM} \end{pmatrix} \tag{17}$$

2.5 Response to point excitation

If a point sinusoidal exciting force $F_p e^{j\omega t}$ is applied at $P(x_p, y_p)$ in the first plane, then

$$\begin{pmatrix} Q_1(x,y) \\ Q_2(y,z) \end{pmatrix} = \begin{pmatrix} F_p \delta(x-x_p)\delta(y-y_p) \\ 0 \end{pmatrix}$$

$$\Rightarrow F_{\ell'm'} = V_{\ell'm'} \cdot \begin{pmatrix} F_p \delta(x-x_p)\delta(y-y_p) \\ 0 \end{pmatrix} = F_p \tilde{\Psi}_{1x\ell'}(x_p)\tilde{\Phi}_{ym'}(y_p);$$

$$\ell' = 1, \cdots, L; \quad m' = 1, \cdots, M.$$

$$\Rightarrow A_{\ell m} = F_p \sum_{\ell'=1}^{L} \sum_{m'=1}^{M} S(\ell, m, \ell', m') \tilde{\Psi}_{1x\ell'}(x_p)\tilde{\Phi}_{ym'}(y_p)$$

$$\Rightarrow \begin{pmatrix} W_1(x,y) \\ W_2(y,z) \end{pmatrix} = F_p \sum_{\ell,m,\ell',m'} S(\ell,m,\ell',m') \tilde{\Psi}_{1x\ell'}(x_p)\tilde{\Phi}_{ym'}(y_p) \begin{pmatrix} \Psi_{1x\ell}(x)\Phi_{ym}(y) \\ \Phi_{ym}(y)\Psi_{2z\ell}(z) \end{pmatrix}. \tag{18}$$

If a receiving point R is at $R(y_R, z_R)$ in the second plane, then the displacement amplitude at R

$$W_2(y_R, z_R) = F_p \sum_{\ell,m,\ell',m'} S(\ell,m,\ell',m') \tilde{\Psi}_{1x\ell'}(x_p)\tilde{\Phi}_{ym'}(y_p)\Phi_{ym}(y_R)\Psi_{2z\ell}(z_R) \tag{19}$$

The transfer mobility from P to R

$$M(1, x_p, y_p; 2, y_R, z_R) = \frac{j\omega W_2(y_R, z_R)}{F_p}$$

$$= j\omega \sum_{\ell,m,\ell',m'} S(\ell,m,\ell',m') \tilde{\Psi}_{1x\ell'}(x_p)\tilde{\Phi}_{ym'}(y_p)\Phi_{ym}(y_R)\Psi_{2z\ell}(z_R) \tag{20}$$

If R coincides with P, then the input mobility at P

$$M(1, x_p, y_p) = j\omega \sum_{\ell, m, \ell', m'} S(\ell, m, \ell', m') \tilde{\Psi}_{1x\ell'}(x_p) \tilde{\Phi}_{ym'}(y_p) \Phi_{ym}(y_p) \Psi_{1x\ell}(x_p) \tag{21}$$

$$= 1 / Z(1, x_p, y_p)$$

where Z $(1, x_p, y_p)$ is the input impedance at P (x_p, y_p) in plane I.

3. A series of T-combination of planes

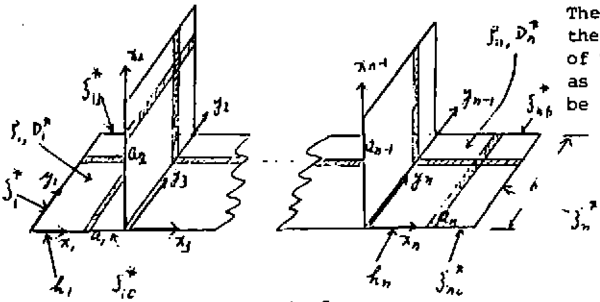


Fig.2.

The bending vibration of the global system of a series of T-combinations of n planes as shown in Fig.2 can also be solved in the same way as discussed in Section 2 for the L-combination. The coordinate function vectors are organized from the eigen modes of the series of T-combinations of beams of unit width and the eigen modes of single beams parallel to the coupled edges (the shaded areas in Fig.2.).

4. Frequency response curves due to point excitation

The frequency response curves of an L-combination of concrete plates (as shown in Fig.1) due to unit force excitation at (3.1) in Plate I are given in Fig.3 (a), (b), (c). The constants of plates and boundary rotational stiffnesses are (all in S.I. Units):

$$a = 5, \ b = 4, \ c = 3; \quad h_1 = h_2 = 0.1; \ D_1^* = D_2^* = 2.2 \times 10^6 (1 + j\eta);$$

$$\frac{\rho_1 h_1}{D_1^*} = \frac{\rho_2 h_2}{D_2^*} = 1.03 \times 10^{-4} (1 - j\eta); \quad (\eta = 1\%, \ 5\%, \ 10\%);$$

all boundary rotational stiffnesses $=100(1+j\,20\%)$. Receiving points are R_1 (2, 2.5) in plate I, R_2(2,2) and R_3 (0.15, 0.15) in Plate II.

The frequency response curves of a series of T-combinations (as shown in Fig.2) of five concrete plates due to unit force excitation at (2.1, 1.5) in Plate I are given in Fig.4 (a), (b), (c). The constants of plates and boundary rotational stiffnesses are (all in S.I. units):

$$a_1 = 3.0\,,\; a_2 = 3.3\,,\; a_3 = 4.5\,,\; a_4 = 4.3\,,\; a_5 = 3.5\,,\; b = 4.0\,;$$

$$h_1 = h_2 = h_3 = h_5 = 0.1\,,\; h_4 = .071\,;$$

$$D_1^* = D_2^* = D_3^* = D_5^* = 2.2\times10^6(1+j\eta)\,,\; D_4^* = 0.778\times10^6(1+j\,10\%)\,;$$

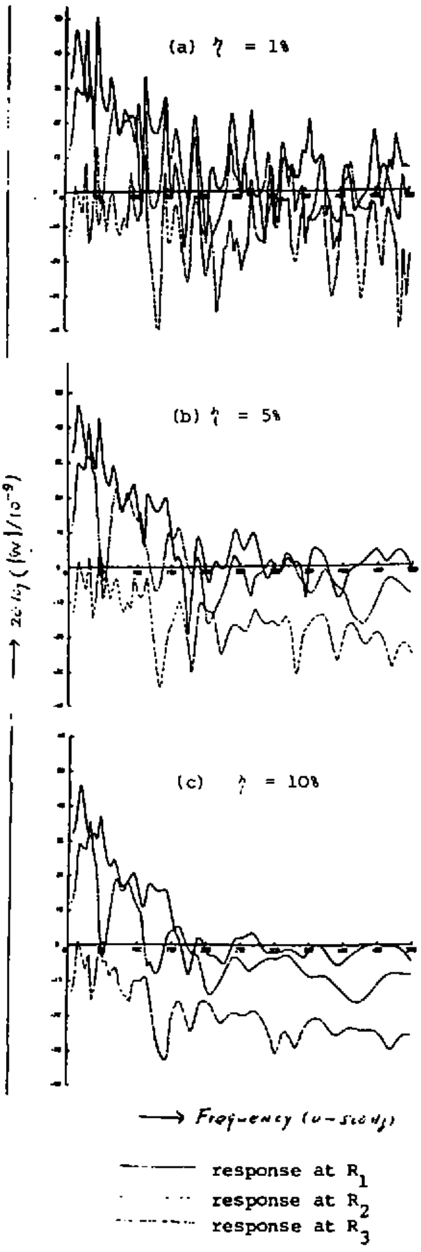
$$\frac{\rho_1 h_1}{D_1^*} = \frac{\rho_2 h_2}{D_2^*} = \frac{\rho_3 h_3}{D_3^*} = \frac{\rho_5 h_5}{D_5^*} = 1.03\times10^{-4}(1-j\eta)\,,\; \frac{\rho_4 h_4}{D_4^*} = 2.06\times10^{-4}(1-j\,10\%)\,;$$

$$\xi_1^* = 10^5\times(1+j\eta)\,,\; \xi_2^* = \xi_3^* = 2\times10^5(1+j\eta)\,,\; \xi_4^* = 0.5\times10^5(1+j\,10\%)\,;$$

$$\xi_{1a}^* = \xi_{2a}^* = \xi_{3a}^* = \xi_{5a}^* = 1.1\times10^5(1+j\eta)\,,\; \xi_{4a}^* = .389\times10^5(1+j\,10\%)\,;$$

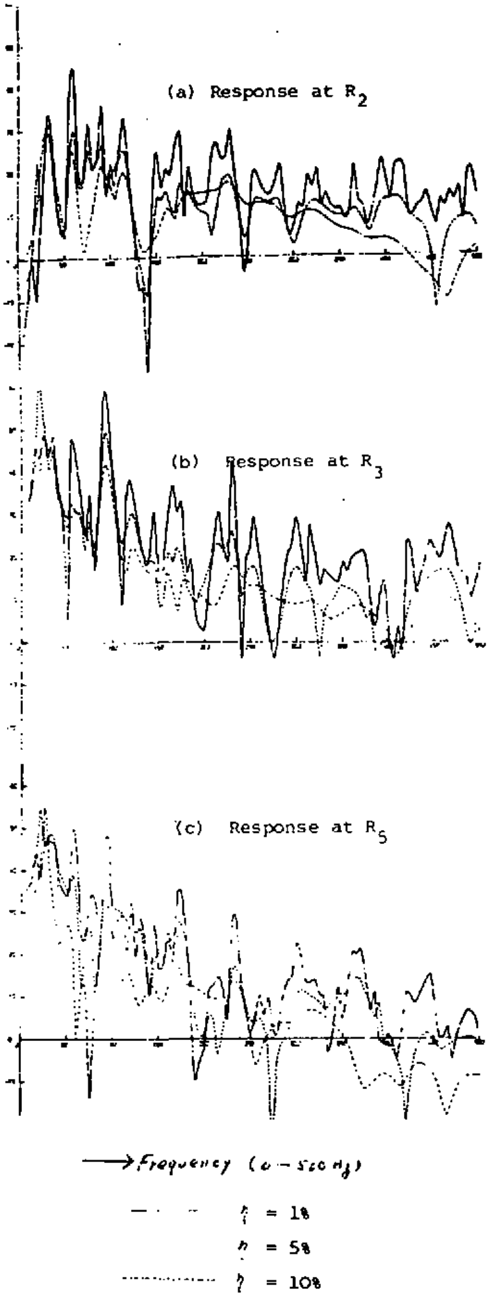
$$\xi_{1b}^* = \xi_{2b}^* = \xi_{3b}^* = \xi_{5b}^* = 2.2\times10^5(1+j\eta)\,,\; \xi_{4b}^* = 0.778\times10^5(1+j\,10\%)\,;$$

($\eta = 1\%,\ 5\%,\ 10\%$). Receiving points R_2(3.1,3.0), R_3(2.5,1.8) and R_5(1.8 2.3) are in plate II, plate III and plate V respectively.



(a) η = 1%
(b) η = 5%
(c) η = 10%
→ 20 log (|w|/10⁻⁹)
→ Frequency (c-sec Hz)
response at R_1
response at R_2
response at R_3

Fig.3



(a) Response at R_2
(b) Response at R_3
(c) Response at R_5
→ Frequency (c-sec Hz)
η = 1%
η = 5%
η = 10%

Fig.4