SOUND RADIATION FROM WALLS AND WINDOWS

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One of the problems often confronting the designer of industrial building is the need to satisfy planning authorities that the noise level emanating from this building will be acceptable at nearby dwellings. Information relating to the directivity of typical building elements is almost non-existent. This paper describes an attempt to remedy this deficiency.

Traditional masonry construction is recognised as providing adequate acoustic insulation for all but the most intense noise sources. With this type of construction problems usually only arise due to sound radiated from acoustic weak points such as windows. Modern industrial buildings tend to consist ot steel frames covered by some kind of lightweight cladding material which is a significant source of sound radiation. Both the windows and cladding can be considered as plates excited into vibration by the sound pressure inside the building.

The differential equation governing the small-amplitude transverse vibration of a rectangular isotropic plate of uniform thickness is

$$D\nabla^4\omega(x,y,t) + \rho h \frac{\partial^2\omega(x,y,t)}{\partial t^2} = q(x,y,t) \qquad (1)$$

where $\omega(x,y,t)$ is displacement function of the plate

$q(x,y,t)$ pressure function applied on the plate

$\nabla^4 = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2$, x, y, coordinates of plate,

a, b are lengths of the edges of the plate (assume a > b)

t, time; ρ density; h thickness; D flexural rigidity,

$D = Eh^3/12(1-\nu^2)$, E Young's modulus, ν Poisson's ratio.

Considering the effect of internal damping, we may introduce the complex flexural rigidity D* instead of D in the governing differential equation,

$$D^* = D(1+j\eta)$$

where η is the loss factor of internal damping of the plate.

The boundary conditi-ns at x = 0 and x = a are:

for a clamped edge, $w = \frac{\partial w}{\partial x} = 0$ (2)

for a simply supported edge, $w = \frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}$ (3)

and for an elastically supported edge,

$$w = 0 \quad \text{and} \quad \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} = \begin{cases} \frac{\epsilon}{a} \frac{\partial w}{\partial x} & \text{at } x = 0 \\ -\frac{\epsilon}{a} \frac{\partial w}{\partial x} & \text{at } x = a \end{cases} \tag{4}$$

where ϵ is a boundary restraint parameter, depending on frequency and the properties of the boundary.

The corresponding boundary conditions for the edges y = 0, y = b may be obtained by interchanging x and y in equations (2) (3) and (4) and by replacing a by b in equation (4).

If $g(x,y,t)$ is uniformly distributed and sinusoidal

$$g(x,y,t) = Q e^{j\omega t} \tag{5}$$

where ω is (circular) frequency, Q is a constant, then $w(x,y,t)$ can be expressed as

$$w(x,y,t) = W(x,y) e^{j\omega t} = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn} X_m(x) Y_n(y) e^{j\omega t} \tag{6}$$

where $W(x,y)$ is displacement amplitude function,

$X_m(x)\, Y_n(y) \quad (m = 1 \cdots\cdots M, \quad n = 1 \cdots\cdots, N)$

are selected coordinate functions satisfying the given boundary conditions of the plate, the coefficients A_{mn} are generalised coordinates.

Since simple polynomials are poor approximations to higher modes of vibrations, in this paper the products of beam characteristic functions satisfying the given boundary conditions are used as coordinate functions. As assumed in (5), the exciting force is uniformly distributed, so only symmetrical beam modes need be considered.

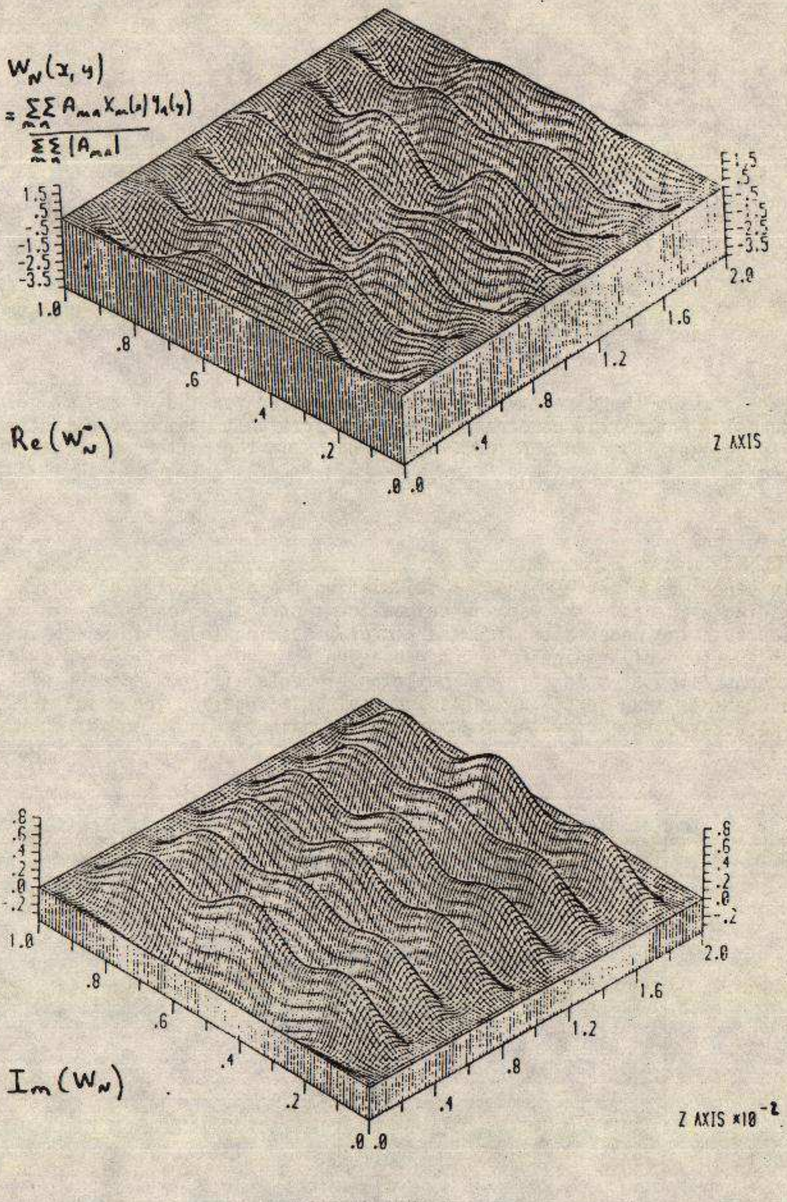
Substituting (5) and (6) into (1) and cancelling the time factor $e^{j\omega t}$ we get

$$D^* \nabla^4 \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn} X_m(x) Y_n(y) - \omega^2 \rho h \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn} X_m(x) Y_n(y) - Q = \mathcal{E}(x,y) \tag{7}$$

where $\mathcal{E}(x,y)$ is an error function. Multiplying both sides of equation (7) by $X_{m'}(x)\, Y_{n'}(y)$, integrating with respect to x and y and applying Galerkin's orthogonality conditions yields a system of MN linear non-homogenous equations from which a final solution to the forced vibration equation can be obtained.

Figure 1 shows the real and imaginary vibration amplitudes calculated for a glazed window excited at a frequency of 1 kHz. The window dimensions were 1m x 2m and thickness 0.01m. The damping coefficient was taken as $\eta = 0.01$ and the four edges were assumed to be clamped.

The acoustic field radiated from a vibrating rectangular plate at a distant point may be calculated by using Rayleigh's approximate formula (see Fig. 2).


$W_N(x,y) = \frac{\sum_m \sum_n A_{mn} X_m(x) Y_n(y)}{\sum_m \sum_n |A_{mn}|}$
$Re(W_N^-)$
Z AXIS
$Im(W_N)$
Z AXIS $\times 10^{-2}$

F.g. 1

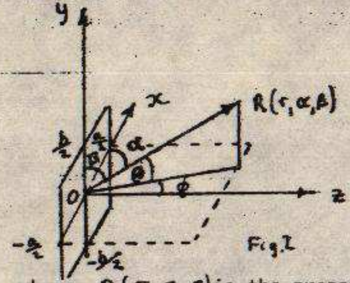


Fig.2

If $r >> a,b$

$$p(r, \alpha, \beta) = j \frac{\rho_o c}{2\pi r} e^{-jkr} \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} V(x,y) e^{jk(x\cos\alpha + y\cos\beta)} dx\,dy \quad (8)$$

where $p(r, \alpha, \beta)$ is the pressure amplitude at point $R(r, \alpha, \beta)$;
k sound wave number in air; r distance between origin O and R
ρ_o and c are density of air and sound velocity in air respectively;
$V(x,y)$ is velocity amplitude function of the vibrating plate and equal to $j\omega W(x,y)$; α, β are angles between OR and x-axis, y-axis respectively.

Figure 3 shows the directivity pattern calculated for the window of Figure 1. For narrow band random noise excitation the same overall trend could be expected although individual troughs in the pattern would tend to be filled in.

Conclusion

This paper describes a method of calculating the directivity of typical building elements. The work described forms part of a long term project which is being undertaken with the aim of providing the building designer with a method of predicting sound radiation from buildings. Directivity patterns calculated for typical building elements will be compared with measurements made on scale models and real buildinga and the results used, if necessary, to modify the theoretical approach.

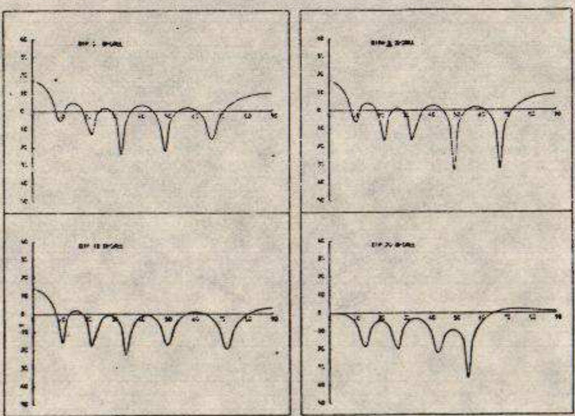


Fig. 3