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DYNAMIC CHARACTERISTICS ANALYSIS OF DISK-COUPLING-SHAFT SYSTEM CONSIDERING DISK ECCENTRICITY AND DISTORTION OF COUPLING

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With the development of heavy industry, elastic coupling plays a more and more important role in internal drive system. The performances of elastic coupling may result in different working condition and efficiency. It will get benefit from studies on coupling dynamics. Applying the results in engineering to solve shear and invalidation because of vibration to make the shafting working more efficient. Researches on coupling mainly concentrate on the bending vibration and torsional vibration. The previous researches show that there is a coupling relationship between the bending vibration and torsional vibration of the shafting system. It is complicated to analyze the model because of the nonlinear vibration mechanics. In order to reduce the difficulty of problem analysis, a simple basic model is usually chosenfor analysis. Based on the analysis of mechanics and the basic theory of elastic mechanics, the system composed of disk and coupling is analyzed. The vibration model of the disc-coupling system is established by Lagrangian equation and vibrational principle, and the differential equations of vibration of the system is deduced. The vibration differential equations are solved by MATLAB, and the internal coupling relation of the vibration of the disc - coupling-shaft system is analyzed in this paper.
Keywords: vibration, disk-coupling-shaft system, Lagrangian equation, coupling dynamics

1. Introduction:

In the engineering practice, the rotating system is dependent on the flexible coupling in most large mechanical equipment. The performance of elastic coupling determines the running efficiency of the machine. The research on the dynamic characteristics of the elastic coupling can help to reduce the fracture and failure of the driving shaft caused by the vibration in the engineering application.

The dynamic characteristics of the coupling are studied by studying the bending vibration and torsional vibration of the shaft coupling system[1]. The results show that there is a coupling relationship between bending vibration and torsional vibration[2], and the simplest coupling model is the Jeffcott Rotor Model. As the bending-torsional coupling vibration involves nonlinear vibration, its variation law and mathematical analysis model are more complex and difficult[3]. In order to reduce the difficulty of these problem, usually a simple basic model is chosen for analysis[4]. Based on the analysis of the basic theories of mechanics and elastic mechanics, the system composed of disk and coupling was analyzed in this paper considering disc eccentricity and distortion of coupling. The vibration model of the disc - coupling system is established by Lagrangian equation and variational principle[5]. The vibration differential equations of the system are deduced and solved by MATLAB, and the internal coupling relation of the vibration of the disc - coupling system is analyzed in this paper.

2. Establishment of vibration differential equations

2.1 The establishment of disc - coupling system model

The simplified model of the disc - coupling system is established by analyzing the elastic couplings in the engineering practice, as shown in Fig. 1:

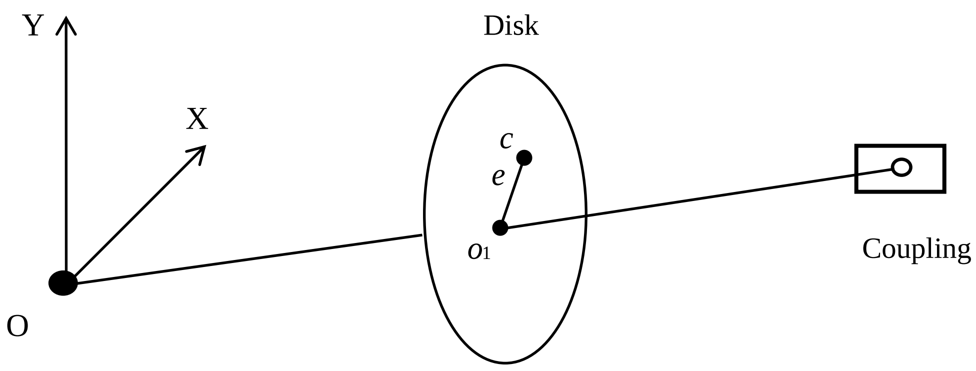


Figure 1:simplified model of the disc - coupling system

In the Fig.1, the rotor is simplified to a single disc whose mass is m_c, and the coupling is simplified as a mass- block whose mass is m_A. The connecting shaft is flexible and isotropic with no mass.

The coordinates of the single disk are shown as below:

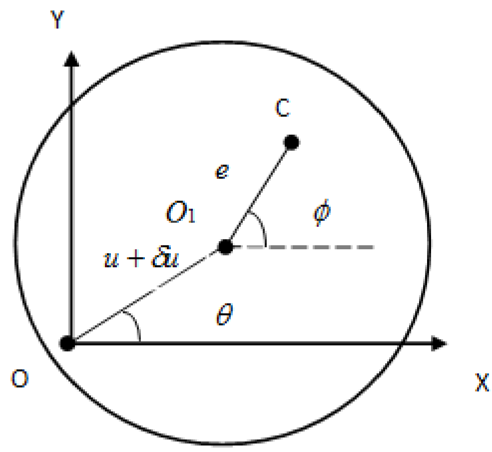


Figure 2:single disk model

In the Fig.2, c is the center mass of the disk, o_1 is the centroid of the disk , o is the center of the vortex motion, ϕ is the angle that the disc has turned and θ is the disc torsion angle. e is the eccentricity of the center of mass to the disc. u is the distance from the center of the disc to the origin, and δu is the initial displacement caused by the gravity of the disc. According to figure.2, the coordinates of the center point can be gotten as follows:

$$y_c = (u+\delta u)\sin\theta + e\sin\phi \tag{1}$$

$$x_c = (u+\delta u)\cos\theta + e\cos\phi \tag{2}$$

The coordinates of the coupling are shown below:

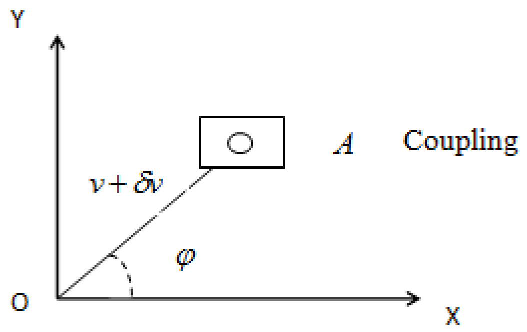


Figure 3:the coordinates of the coupling

In the Fig.3, o is the vortex center, v is the distance from the coupling to the origin, δv is the initial displacement caused by the gravity of the coupling. ϕ is the torsion angle of the coupling. According to the coordinate system, the position coordinates of the coupling can be gotten as follows:

$$x_A = (v+\delta v)\cos\varphi \tag{3}$$

$$y_A = (v+\delta v)\sin\varphi \tag{4}$$

2.2 Kinetic energy of the system

According to the coordinate system established above, the velocity of the disk center of mass and the coupling is expressed by generalized coordinates, in which the velocity of the disk center of mass are:

$$\dot{x}_c = \dot{u}\cos(\theta) - (u+\delta u)\sin(\theta)\dot{\theta} - e\sin(\phi)\dot{\phi} \tag{5}$$

$$\dot{y}_c = \dot{u}\sin(\theta) + (u+\delta u)\cos(\theta)\dot{\theta} + e\cos(\phi)\dot{\phi} \tag{6}$$

In this case, the linear velocity of the disk center of mass is:

$$V_c^2 = \dot{x}_c^2 + \dot{y}_c^2 \tag{7}$$

$$= \dot{u}^2 + (u+\delta u)^2\dot{\theta}^2 + e^2\dot{\phi}^2 + 2\dot{u}\dot{\phi}e\sin(\theta-\phi) + 2\dot{\theta}\dot{\phi}(u+\delta u)e\cos(\theta-\phi) \tag{8}$$

For couplings, the speeds in the X and Y directions are:

$$\dot{x}_A = \dot{v}\cos(\varphi) - (v+\delta v)\sin(\varphi)\dot{\varphi} \tag{9}$$

$$\dot{y}_A = \dot{v}\sin(\varphi) + (v+\delta v)\cos(\varphi)\dot{\varphi}^2 \tag{10}$$

The linear speed of the coupling is:

$$V_A^2 = \dot{x}_A^2 + \dot{y}_A^2$$

$$= \dot{v}^2 + (v+\delta v)^2\dot{\varphi}^2 \tag{11}$$

The translational kinetic energy of the disk is:

$$T_G = \frac{1}{2}[\dot{u}^2 + (u+\delta u)^2\dot{\theta}^2 + e^2\dot{\phi}^2 + 2\dot{u}\dot{\phi}e\sin(\theta-\phi) + 2\dot{\theta}\dot{\phi}(u+\delta u)e\cos(\theta-\phi)] \tag{12}$$

The rotational kinetic energy of the disk is:

$$T_r = \frac{1}{2}J_c\dot{\phi}^2 \tag{13}$$

The kinetic energy of the coupling is:

$$T_A = \frac{1}{2}m_A V_A^2$$

$$= \frac{1}{2}m_A(\dot{v}^2 + (v+\delta v)^2\dot{\varphi}^2) \tag{14}$$

In this case, the whole kinetic energy of the system is:

$$T = T_A + T_c \tag{15}$$

$$T=\frac{1}{2}m_A(\dot{v}^2+(v+\delta v)^2\dot{\varphi}^2)+\frac{1}{2}m_c[(\dot{u}^2+(u+\delta u)^2\dot{\theta}^2+e^2\dot{\phi}^2$$
$$+2\dot{u}\dot{\phi}e\sin(\theta-\phi)+2\dot{\theta}\dot{\phi}(u+\delta u)e\cos(\theta-\phi)]+\frac{1}{2}J_c\dot{\phi}^2 \tag{16}$$

2.3 Potential energy of the system

In this paper, we considered that the distortion of the coupling directly affects the change of the center of the disk. Therefore, the bending deformation and torsional deformation of the shaft system must be considered with the effect of the distortion.

The bending displacement of the shaft system is:

$$\varepsilon=[(u+\delta u)\cos\theta-(v+\delta v)\cos\varphi]^2+[(u+\delta u)\sin\theta-(v+\delta v)\sin\varphi)]^2 \tag{17}$$

The torsional displacement of the shaft system is:

$$\psi=\theta-\varphi \tag{18}$$

For the system, its potential energy can be expressed as:

$$U=U_1+U_2+U_3+m_cg(e\sin\phi+u\sin\theta)+m_Agv\sin\varphi \tag{19}$$

In this case, U_1 is the energy of bending deformation:

$$U_1=\frac{1}{2}k\{[(u+\delta u)\cos\theta-(v+\delta v)\cos\varphi]^2+[(u+\delta u)\sin\theta-(v+\delta v)\sin\varphi]^2]\} \tag{20}$$

U_2 is the energy of torsional deformation:

$$U_2=\frac{1}{2}k_t(\theta-\varphi)^2 \tag{21}$$

U_3 is the elastic potential energy of the coupling:

$$U_3=\frac{1}{2}k_u(u+\delta u)^2+\frac{1}{2}k_v(v+\delta v)^2 \tag{22}$$

In the above formula, k is the bending stiffness, k_t is the torsional stiffness, k_u, k_v are the equivalent stiffness of the disk and coupling.

According to the relationship between initial deformation and gravity, the overall potential energy of the system can be written as:

$$U=\frac{1}{2}k\{[(u+\delta u)\cos\theta-(v+\delta v)\cos\phi]^2+[(u+\delta u)\sin\theta-(v+\delta v)\sin\phi]^2\}$$
$$+\frac{1}{2}k_t(\theta-\phi)^2+\frac{1}{2}k_uu^2+\frac{1}{2}k_vv^2+m_cge\sin\phi \tag{23}$$

2.4 Generalized force of the system

When the damping-force of the system is taken into account, the generalized force can be expressed as follows:

$$\begin{aligned}
Q_u&=-c\dot{u}+f_u\\
Q_\theta&=-c_\theta\dot{\theta}+M_\theta\\
Q_\phi&=-c_\phi\dot{\phi}+M_\phi\\
Q_v&=-c\dot{v}+f_v\\
Q_\varphi&=-c_\varphi\dot{\varphi}+M_\varphi
\end{aligned} \tag{24}$$

c is the translational damping, $c_\theta, c_\varphi, c_\phi$ is the rotation damping.

2.5 The Establishment of the Vibration Differential Equations

In the following, each parameter variables are replaced into the Lagrangian equation.

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{u}})-\frac{\partial T}{\partial u}+\frac{\partial U}{\partial u}=Q_u \tag{25}$$

In this case,

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{u}})=m_c\ddot{u}+m_c\ddot{\phi}e\sin(\theta-\phi)-m_c\dot{\phi}^2e\cos(\theta-\phi)+m_c\dot{\theta}\dot{\phi}e\cos(\theta-\phi) \tag{26}$$

$$\frac{\partial T}{\partial u}=m_c(u+\delta u)\dot{\theta}^2+m_c\dot{\theta}\dot{\phi}e\cos(\theta-\phi) \tag{27}$$

$$\frac{\partial U}{\partial u}=k[(u+\delta u)\cos\theta-(v+\delta v)\cos\varphi]\cos\theta+k[(u+\delta u)\sin\theta-(v+\delta v)\sin\varphi]\sin\theta+k_u u \tag{28}$$

For u , the Lagrangian equation can be organized as follows:

$$\begin{aligned}&m_c\ddot{u}+m_c\ddot{\varphi}e\sin(\theta-\varphi)-m_c\dot{\varphi}^2e\cos(\theta-\varphi)-m_c(u+\delta u)\dot{\theta}^2+c\dot{u}+k_u u\\&+k[(u+\delta u)\cos\theta-(v+\delta v)\cos\phi]\cos\theta+k[(u+\delta u)\sin\theta-(v+\delta v)\sin\phi]\sin\theta=f_u\end{aligned} \tag{29}$$

For θ :

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{\theta}})-\frac{\partial T}{\partial \theta}+\frac{\partial U}{\partial \theta}=Q_\theta \tag{30}$$

So the Lagrangian equation can be organized as follows:

$$\begin{aligned}&2m_c(u+\delta u)\dot{u}\dot{\theta}+m_c(u+\delta u)^2\ddot{\theta}+m_c\ddot{\phi}(u+\delta u)e\cos(\theta-\phi)+m_c\dot{\phi}^2(u+\delta u)e\sin(\theta-\phi)+c_\theta\dot{\theta}\\&=k[(u+\delta u)\cos\theta-(v+\delta v)\cos\varphi](u+\delta u)\sin\theta-k[(u+\delta u)\sin\theta-(v+\delta v)\sin\varphi](u+\delta u)\cos\theta\\&-k_t(\theta-\varphi)+M_\theta\end{aligned} \tag{31}$$

For ϕ :

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{\phi}})-\frac{\partial T}{\partial \phi}+\frac{\partial U}{\partial \phi}=Q_\phi \tag{32}$$

So the Lagrangian equation can be organized as follows:

$$\begin{aligned}&m_ce^2\ddot{\phi}+m_c\ddot{u}e\sin(\theta-\phi)+2m_c\dot{u}\dot{\theta}e\cos(\theta-\phi)+m_c\ddot{\theta}(u+\delta u)e\cos(\theta-\phi)\\&-m_c\dot{\theta}^2(u+\delta u)e\sin(\theta-\phi)+J_c\ddot{\phi}+c_\phi\dot{\phi}=-m_cge\cos\phi+M_\phi\end{aligned} \tag{33}$$

For v :

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{v}})-\frac{\partial T}{\partial v}+\frac{\partial U}{\partial v}=Q_v \tag{34}$$

So the Lagrangian equation can be organized as follows:

$$m_A\ddot{v}-m_A(v+\delta v)\dot{\varphi}+c\dot{v}+k(v+\delta v)-k(u+\delta u)\cos(\theta-\phi)=-k_v v+f_v \tag{35}$$

For φ :

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{\varphi}})-\frac{\partial T}{\partial \varphi}+\frac{\partial U}{\partial \varphi}=Q_\varphi \tag{36}$$

So the Lagrangian equation can be organized as follows:

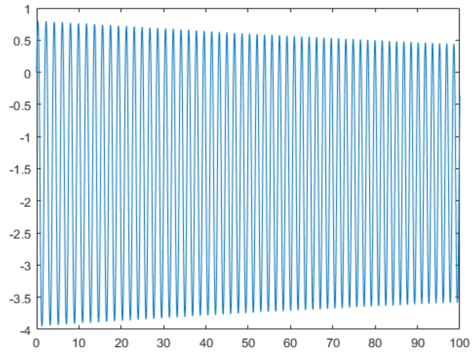
$$2m_A(v+\delta v)\dot{v}\dot{\varphi}+m_A(v+\delta v)^2\ddot{\varphi}+k(u+\delta u)(v+\delta v)\sin(\varphi-\theta)-k_t(\theta-\varphi)+c_\varphi\dot{\varphi}=M_\varphi \tag{37}$$

3. Result Analysis of Equations

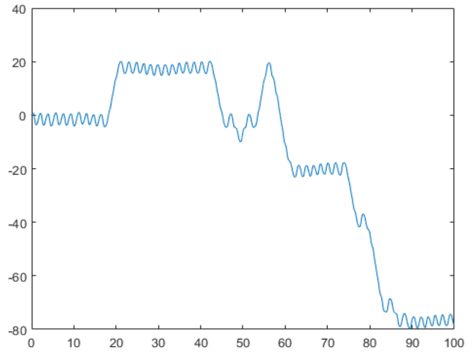
3.1 Solving the equation

For the differential equations of vibration established above, the numerical solution is gotten by using the Runge-Kutta algorithm in MATLAB. Obtained the response of free vibration under the condition of five exciations (different frequency) at the same time:

The vibration response of ϕ

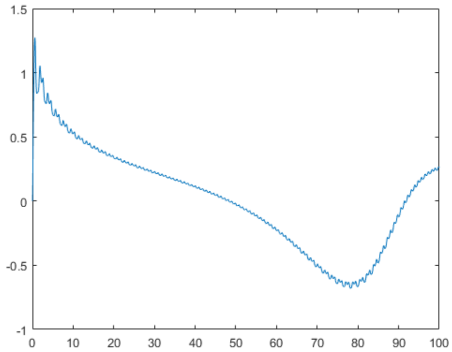


a(1)

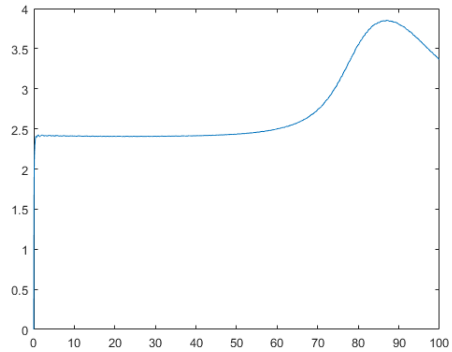


a(2)

The vibration response of θ

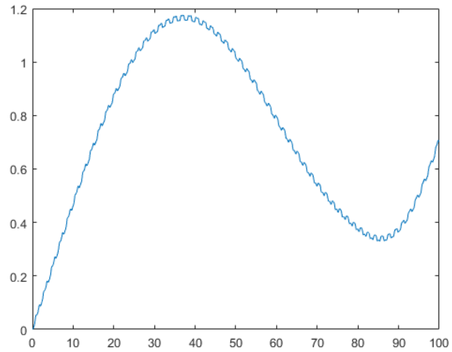


b(1)

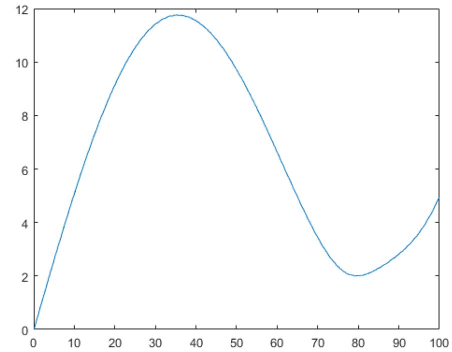


b(2)

The vibration response of u

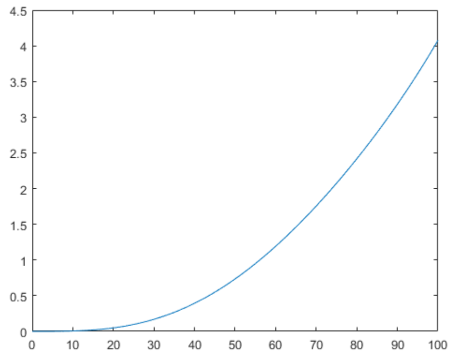


c(1)

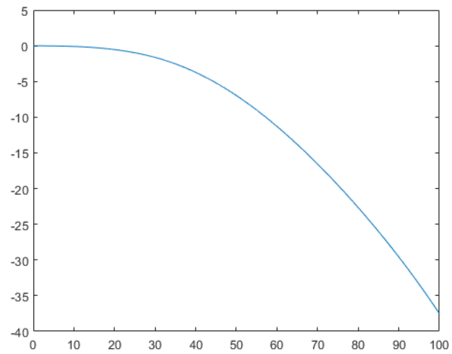


c(2)

The vibration response of v

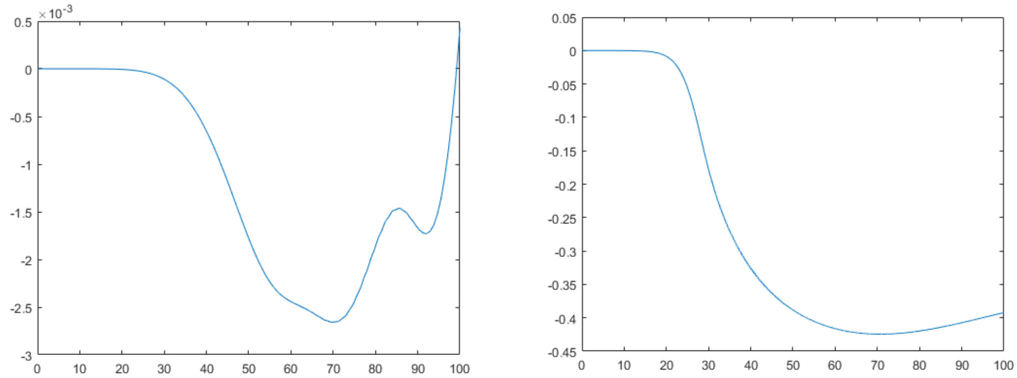


d(1)



d(2)

The vibration response of φ



e(1) e(2)

Figure 4:The response of free vibration(left) and forced vibration(right)

3.2 Results analysis

The correlation between the responses is analyzed in the numerical aspect, and the coupling relationship of these five responses is quantitatively analyzed. The corresponding correlation obtained by MATLAB as shown in the following table. (For the vibration response of φ, choose to ignore it and other responses between the coupling because of its tiny response):

Table 1:The correlation coefficient of the response

free	Φ&θ	Φ&u	Φ&v	θ&u	θ&v	u&v
correlation	0.3024	0.6567	0.5015	0.4917	0.5687	0.8433
Delay(s)	48.4375	0	0	36.5087	11.3743	40.9442
exciation						
correlation	0.9739	0.9535	0.9992	0.9357	0.9839	0.9488
Delay(s)	0	0	0	0	0	0

Through the analysis of the data in the table, it can be seen that the correlation between the responses is low and the degree of coupling is not high in the case of free vibration without external excitation. And it can be seen that there is a delay between the coupling of some responses. But when five (different frequency) exciations applied to the system, the correlation is significantly enhanced. It's shows that the degree of coupling is high. And there is no delay between the coupling of these responses.

4. Conclusions

To sum it up, when the disc eccentricity and distortion of coupling is taken into consideration , there is coupling between the response. And the relationship of these response is shown above. When the five excitation in different frequency applied to the system, the degree of the coupling is relatively higher which proves that there are affects among each response of the system. To reduce the vibration amplitude of a certain response, the whole analysis of the system needs to be carried out.

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