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A SERIES OF VARIABLE STEP-SIZE FXLMS ALGORITHMS FOR NARROWBAND ACTIVE NOISE CONTROL

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In this paper, we propose a series of variable step-size (VSS) filtered-x LMS algorithms (VSS-FXLMS) for narrowband active noise control (ANC, NANC). They are computationally efficient and present improved stability and performance as compared with two existing VSS-FXLMS algorithms developed recently for the NANC. The VSS update terms of the proposed algorithms are positive functions of the residual noise. Extensive simulations with synthetic/real secondary paths and primary noises are conducted to show the performance advantages of the proposed VSS-FXLMS algorithms over their counterparts.

Keywords: Active noise control (ANC), narrowband ANC (NANC), variable step-size FXLMS (VSS-FXLMS), tracking capability, stability.

1. Introduction

In the last two decades, a great deal of attention has been paid to the development of practical and effective active noise control (ANC) systems for real-life applications [1, 2]. The ANC systems may be roughly divided into two types according to the frequency characteristics of the targeted noise signals. The first type is called broadband ANC (BANC) that is used to suppress noise signals whose frequency distribution spans a wide and continuous range in the frequency domain. The second type is called narrowband ANC (NANC) which is designated to mitigate noise signals that contain a single or multiple frequency components which are discretely spaced in the frequency domain [2]. In this work, we focus on the second type.

The NANC systems may be loosely clarified into two kinds according to their controller characteristics. If the controller is realized by a first-order FIR filter or magnitude/phase adjuster (MPA), the system may be called FIR-based NANC system. If the controller is a linear combiner (LC) with a cosine and a sine reference wave, the control filter weights are actually two discrete Fourier coefficients (DFC), and the system may be called DFC-based NANC system [3, 2]. The second kind of NANC system is considered in this work.

In Fig. 1, a typical DFC-based NANC system is depicted [2, 3]. The frequencies of the reference cosine and sine waves $\{x_{a_i}(n) = \cos(\omega_i n), x_{b_i}(n) = \sin(\omega_i n)\}_{i=1}^{q}$ are specified by virtue of a synchronization signal acquired usually by a nonacoustic sensor like tachometer. Here, q indicates the number of frequencies targeted by the NANC system. The control filter weights $\{\hat{a}_i(n), \hat{b}_i(n)\}$ are DFCs of the ith individual secondary source $y_i(n)$ for the ith frequency channel. The secondary source of the NANC system is expressed by

$$y(n) = \sum_{i=1}^{q} y_i(n) = \sum_{i=1}^{q} \left[\hat{a}_i(n)x_{a_i}(n) + \hat{b}_i(n)x_{b_i}(n)\right]. \quad (1)$$

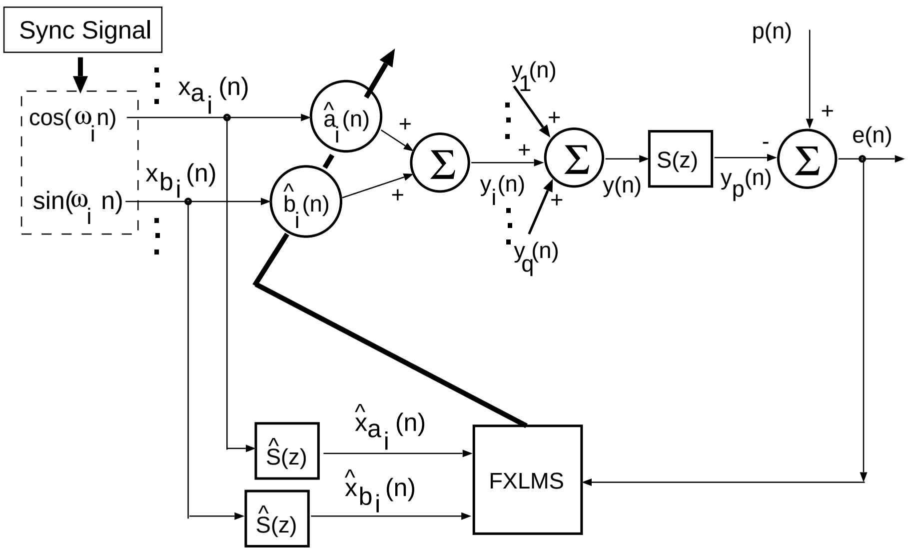

Fig. 1 Block diagram of a typical DFC-based NANC system

$S(z)$ shows the discrete transfer function of the secondary path which includes the loudspeaker, the microphone that captures the residual noise $e(n)$, and the space between them. The secondary path may be expressed as an FIR filter:

$$S(z) \;=\; \sum_{m=0}^{M-1} s_m z^{-m} \tag{2}$$

where M is the length, and $\{s_m\}_{m=0}^{M-1}$ are the FIR filter coefficients of the secondary path. This secondary path is usually identified in advance by some system identification technique like Wiener filter, and is indicated by $\hat{S}(z)$.

$$\hat{S}(z) \;=\; \sum_{j=0}^{\hat{M}-1} \hat{s}_j z^{-j} \tag{3}$$

where $\hat{M}$ is the length, and $\{\hat{s}_j\}_{j=0}^{\hat{M}-1}$ are the FIR coefficients of an estimate of the secondary path.

The primary noise signal $p(n)$ is given by

$$p(n) \;=\; \sum_{i=1}^{q} [a_{p,i}(n) x_{a_i}(n) + b_{p,i}(n) x_{b_i}(n)] + v_p(n) \tag{4}$$

where $\{a_{p,i}, b_{p,i}\}_{i=1}^{q}$ are the DFCs of the frequency components residing in the primary noise signal, $v_p(n)$ is a zero-mean additive random noise with variance σ_p^2. This additive noise embodies all random factors related to the targeted noise. It is unattended in the NANC system, but affects the performance of the overall system in a statistical way.

The control filter weights of the system are updated by a filtered-x LMS (FXLMS) algorithm

$$\hat{a}_i(n+1) \;=\; \hat{a}_i(n) + \mu e(n) \hat{x}_{a_i}(n) \tag{5}$$

$$\hat{b}_i(n+1) \;=\; \hat{b}_i(n) + \mu e(n) \hat{x}_{b_i}(n) \tag{6}$$

where μ is a uniform step size for all frequency channels, $\hat{x}_{a_i}(n)$ and $\hat{x}_{b_i}(n)$ are reference waves filtered by the estimate of the secondary path $\hat{S}(z)$

$$\hat{x}_{a_i}(n) \;=\; \sum_{j=0}^{\hat{M}-1} \hat{s}_j x_{a_i}(n-j) \tag{7}$$

$$\hat{x}_{b_i}(n) = \sum_{j=0}^{\hat{M}-1} \hat{s}_j x_{b_i}(n-j). \tag{8}$$

The DFC update of this FXLMS algorithm only requires three multiplications for one frequency. Therefore, it is very efficient. In fact, it also works well as long as the secondary-path estimate $\hat{S}(z)$ is close to its truth. That is why the FXLMS boasts of being the most popular algorithm in ANC. However, in some real applications the performance of the FXLMS may be inadequate. For example, if a NANC system is applied to noises generated by large-scale cutting machines, it is desirable that the system converges as fast as possible and presents nice stability as well as designed steady-state noise reduction performance, even in the presence of harsh load fluctuation. In such a situation, the FXLMS is not good enough. One may consider to use the FXRLS algorithm to fulfill the performance requirements, but the FXRLS costs much more than the FXLMS despite that its tracking capability is only comparable to that of the FXLMS.

To remedy the FXLMS and FXRLS, we recently proposed two variable step-size (VSS) FXLMS (VSS-FXLMS) algorithms [4, 5], after carefully investigating existing typical VSS-LMS algorithms developed in the context of adaptive FIR filtering [6]-[11]. These two previous algorithms have advantages over the conventional FXLMS, but also possess their own drawbacks.

In this work, we propose a series of VSS-FXLMS algorithms which are more stable than the VSS-FXLMS developed in [4] and includes the algorithm shown in [5] as a similar case. Extensive simulations are conducted to prove their improved stability and promising applicability.

2. The proposed VSS-FXLMS algorithms

First, let's have a look at our previous VSS-FXLMS algorithms. Their DFC update equations are the same as in the FXLMS except that their step sizes are variable and are updated as follows:

$$\mu(n+1) = \xi\mu(n) + \eta e(n)\hat{x}_{a_1}(n)e(n-1)\hat{x}_{a_1}(n-1) \quad [4] \tag{9}$$

$$\mu(n+1) = \xi\mu(n) + \eta e^2(n)e^2(n-1) \quad [5] \tag{10}$$

where ξ and η are user parameters. Parameter ξ is less than 1.0, but very close to 1.0, such as 0.999, 0.9995, etc., η usually takes a very small positive value. The combination of ξ and η defines the properties of the VSS.

If the user parameters in (9) are well chosen, the VSS-FXLMS algorithm presents significantly improved performance as compared to the conventional FXLMS algorithm [4]. However, this VSS-FXLMS algorithm has an inherent problem of its own. The second term in the right-hand side (RHS) of (9) is the VSS update term. The VSS may drift into the minus zone despite the ability of the overall algorithm to avoid such an unfavorable situation, because its update term has a risk of becoming negative during the adaptation process. It has been found via simulations that, if the user parameters in (9) are not properly set the system will work very poorly or even diverge. In-depth analysis of the algorithm has not been done, but extensive simulations have revealed these unfavorable properties of the algorithm.

Obviously, the VSS update term in the RHS of (10) is always positive, which implies that the trouble with (9) will not happen as long as the user parameters are properly set. At steady state, the mean value of $e^2(n)e^2(n-1)$ is approximately σ_p^4, as the dominant component within $e(n)$ will be $v_p(n)$. As $E[e^4(n)]$ reduces to $3\sigma_p^4$ at steady state, $e^2(n)e^2(n-1)$ behaves like $\frac{1}{3}e^4(n)$ in a statistical sense. The performance of the algorithm using this VSS is comparable to the conventional one [6], but it requires less computational effort [5].

Inspired by the above observations, we propose the following general VSS update equation

$$\mu(n+1) = \xi\mu(n) + \eta|e(n)|^r, \; r = 1, 2, \cdots \tag{11}$$

which forms a series of VSS-FXLMS algorithms. Apparently, the VSS is always positive, resulting in stable system performance when the user parameters are well selected. If r is set to 1, the VSS update term will need only a single multiplication. Three multiplications are required for $r = 2$. For $r = 3$ and 4, the number of multiplications will be four. Therefore, the proposed VSS update is quite slim. The problem is how the user parameter r affects the performance of the resultant algorithm.

3. Simulation results

To confirm the superior performance of the proposed VSS-FXLMS algorithm, extensive simulations have been conducted that synthetic/real secondary paths and primary noise signals are used. Here, only some representative simulation results are provided.

In all simulations, unless otherwise specified, the primary noise $p(n)$ consists of three frequency components ($q = 3, \omega_1 = 0.1\pi, \omega_2 = 0.2\pi, \omega_3 = 0.3\pi$). Their DFCs are $a_1 = 2.0, b_1 = -1.0, a_2 = 1.0, b_2 = -0.5, a_3 = 0.5$, and $b_3 = 0.1$. The variance of the additive noise $v_p(n)$ is 0.1. Two secondary paths were used. The first one is an FIR filter or lowpass filter of length $M = 21$ generated by a Matlab function fir1 (cutoff frequency $= 0.4\pi$). Its estimate with length $\hat{M} = 31$ was obtained in advance via the LMS algorithm, where the step size was $\mu_s = 0.0025$, the input white noise variance was set to 1.0, and the additive observation noise was set to be 0.01% of the clean observation data in power (i.e., SNR=40 [dB]). The second one is a real IIR filter given in [2]. An FIR filter of length $\hat{M} = 32$ was used to approximate the IIR filter, and was identified in the same way the first secondary-path estimate was obtained. A hundred (100) independent runs were done for each case to compare ANC systems with FXLMS and VSS-FXLMS algorithms in terms of MSE between the control filter weights (DFCs) and their optimum values $\{\hat{a}_{i,opt}, \hat{b}_{i,opt}\}_{i=1}^{q}$ given in [3]. User parameters for all VSS-FXLMS algorithms were adjusted very carefully such that all of them present similar steady- state performance.

3.1 Case A: An FIR secondary path

The conventional FXLMS, the VSS-FXLMS by Huang et al. [4], the VSS-FXLMS by Xiao et al. [5], and the proposed algorithm with $r = 4$ are plotted in Fig. 2.

It may be clearly seen in Fig. 2 that the three VSS-FXLMS algorithms converge similarly in the early stage of adaptation, much faster than the conventional FXLMS. However, the VSS-FXLMS algorithm by Huang et al. [4] starts rising when it approaches its steady state, showing the risk of diverging or possibly reaching some higher MSE level as the iteration goes on. This case lets us have a look at the poor stability of the VSS-FXLMS algorithm by Huang et al. [4].

In Fig. 3, the simulation results for the proposed algorithm with $r = 1, 2, \cdots, 6$ are provided, where the primary noise signal was deliberately made nonstationary by switching its DFCs signs to the opposite just in the middle of the adaptation process.

From Fig. 3, one may readily find that the proposed algorithm converges faster in the early stage as r becomes larger in both the first and the second half. However, when r is larger than 3, the algorithm actually becomes quite sluggish and takes longer to reach the steady state as compared to the case with $r < 3$. Therefore, it is better for us to set $r \leq 3$ when applying the proposed algorithm.

3.2 Case B: A real IIR secondary path

In this case, a real IIR filter given in [2] was used. Its estimate is an FIR filter with $\hat{M}(= 32)$ coefficients. It was obtained also via the LMS algorithm in exactly the same way the FIR secondary-path estimate was identified. The primary noise signal included in this case was generated by a large-scale cutting machine. It is nonstationary, with the first and second half being recorded when the machine was running at 1400 rpm and 1600 rpm, respectively. The frequency analysis of the 1st and 2nd half of the primary noise is shown in Fig. 4. In the first half, five (5) frequencies (0.0804π,

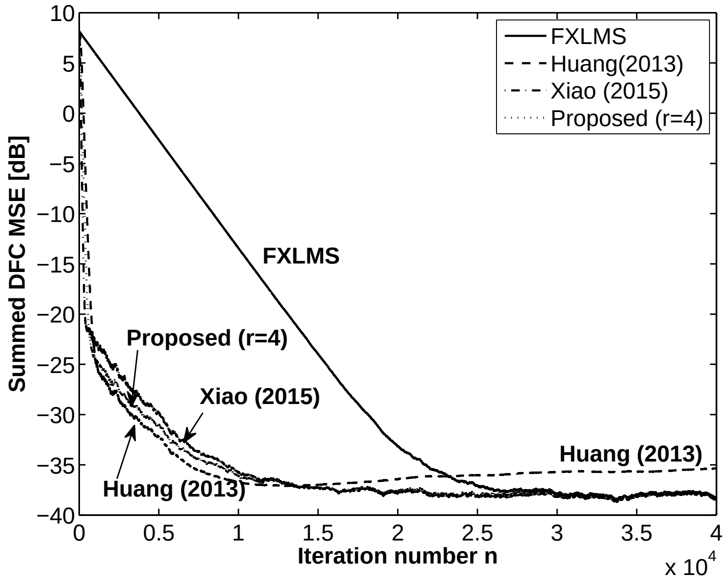

Fig. 2 Comparison among three VSS-FXLMS algorithms (conventional: $\mu = 0.0005$, VSS-FXLMS (Huang (2013), [4]) : $\xi = 0.9995, \eta = 3.5 \times 10^{-5}$, VSS-FXLMS (Xiao (2015), [5]) : $\xi = 0.9995, \eta = 2.0 \times 10^{-5}$, Proposed: $r = 4, \xi = 0.9995, \eta = 2.28 \times 10^{-6}$).

0.1609π, 0.2414π, 0.3218π, 0.4024π or 411.6480 Hz, 823.8080 Hz, 1235.9680 Hz, 1647.6160 Hz, 2060.2880 Hz) were targeted. In the second half, four (4) frequencies (0.0902π, 0.1807π, 0.2710π, 0.3614π, or 461.8240 Hz, 925.1840 Hz, 1387.5200 Hz, 1850.3680 Hz) were the subjects of the NANC. The proposed algorithm with $r = 1, 2,$ and 3 was applied to this real nonstationary primary noise signal.

The residual noises produced by the FXLMS and our proposed algorithm with $r = 1, 2, 3$ are plotted in Fig. 5. As noted with ease in Fig. 5, the proposed algorithm converges faster and indicates more noise reduction as r gets larger in both the first and second half, showing its effectiveness and applicability.

Furthermore, in the first half, the primary noise was reduced by approximately 11 dB by all algorithms, but the proposed algorithm with $r = 3$ showed the fastest convergence. The primary noise in the second half was reduced by 7 and 11 [dB] by the FXLMS and the proposed algorithm, respectively, and the proposed algorithm also presented the faster convergence.

4. Conclusions

In this paper, a series of VSS-FXLMS algorithms have been proposed for NANC. They enjoy improved stability as compared to the first VSS-FXLMS algorithm dedicated to the NANC [4]. They also present overall performance better than that of another previous VSS-FXLMS algorithm [5]. A lot of simulations have been conducted to prove our claim regarding the proposed VSS-FXLMS algorithms. Implementing the proposed algorithms in real applications is the first future topic. In-depth analysis of them is technically demanding, which is expected to provide a very useful piece of information on their statistical behaviors. This is the second topic for further research.

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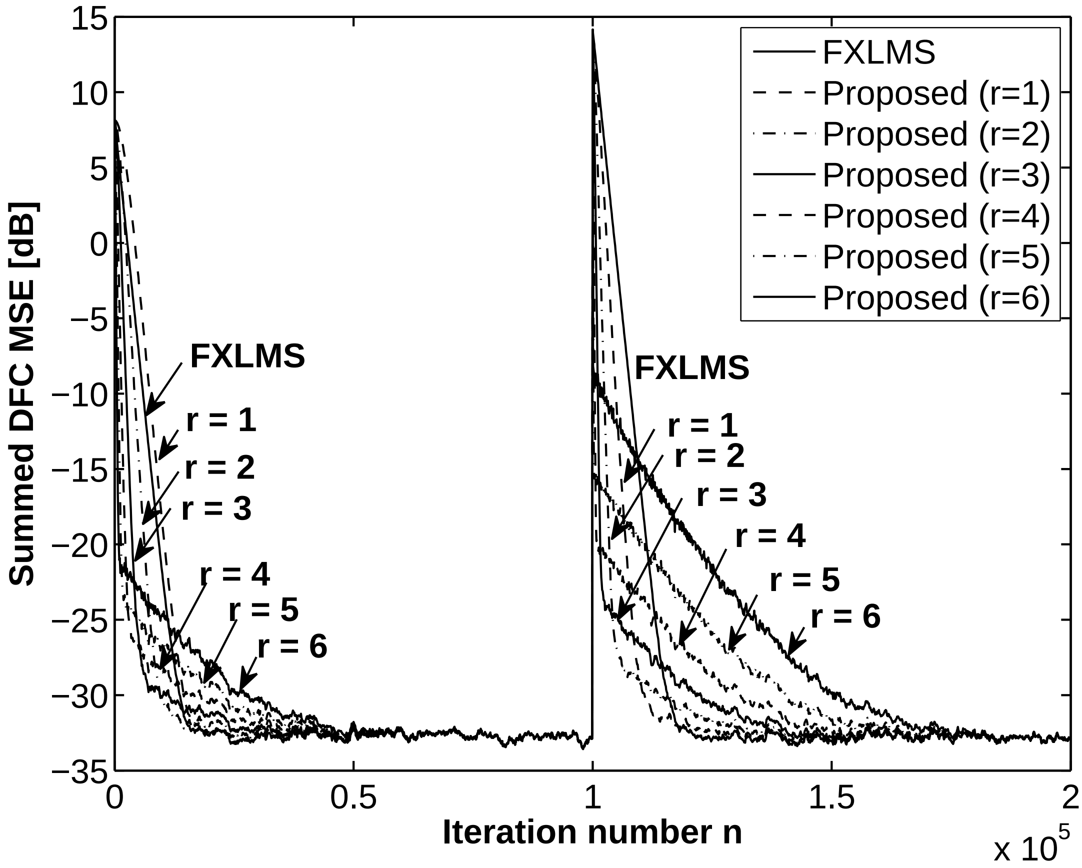

Fig. 3 Summed DFC MSE [dB] of the proposed VSS-FXLMS algorithm for different $r(1, 2, \cdots, 6)$ in the presence of a nonstationary synthetic primary noise (FXLMS: $\mu = 0.0005$; Proposed $(r = 1)$: $\xi = 0.9995, \eta = 9.50 \times 10^{-7}$; Proposed $(r = 2)$: $\xi = 0.9995, \eta = 2.28 \times 10^{-6}$; Proposed $(r = 3)$: $\xi = 0.9995, \eta = 4.30 \times 10^{-6}$; Proposed $(r = 4)$: $\xi = 0.9995, \eta = 7.00 \times 10^{-6}$; Proposed $(r = 5)$: $\xi = 0.9995, \eta = 1.01 \times 10^{-5}$; Proposed $(r = 6)$: $\xi = 0.9995, \eta = 1.28 \times 10^{-4}$, 100 runs).

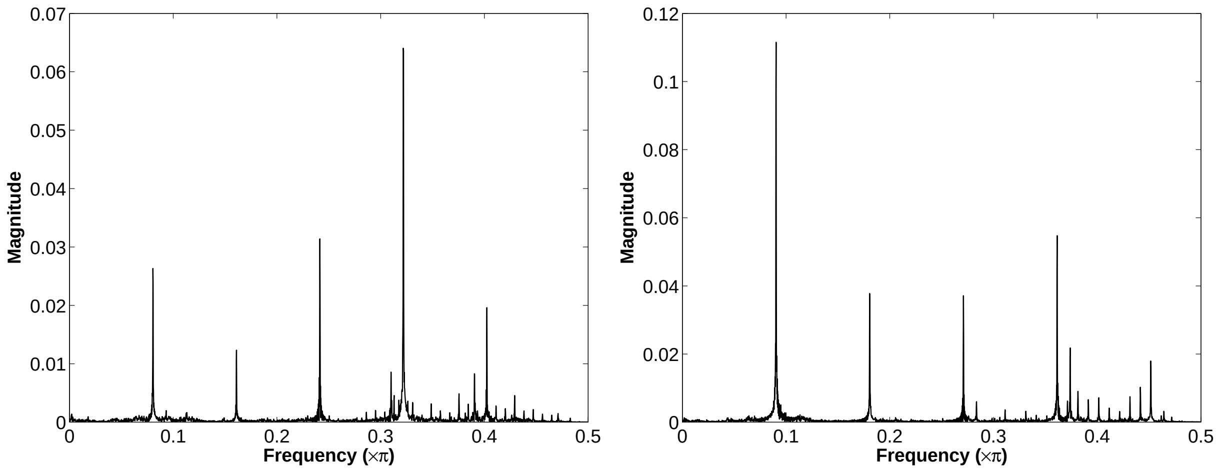

Fig. 4 Frequency analysis of the primary noise (left: 1st half, right: 2nd half) generated by a large-scale cutting machine.

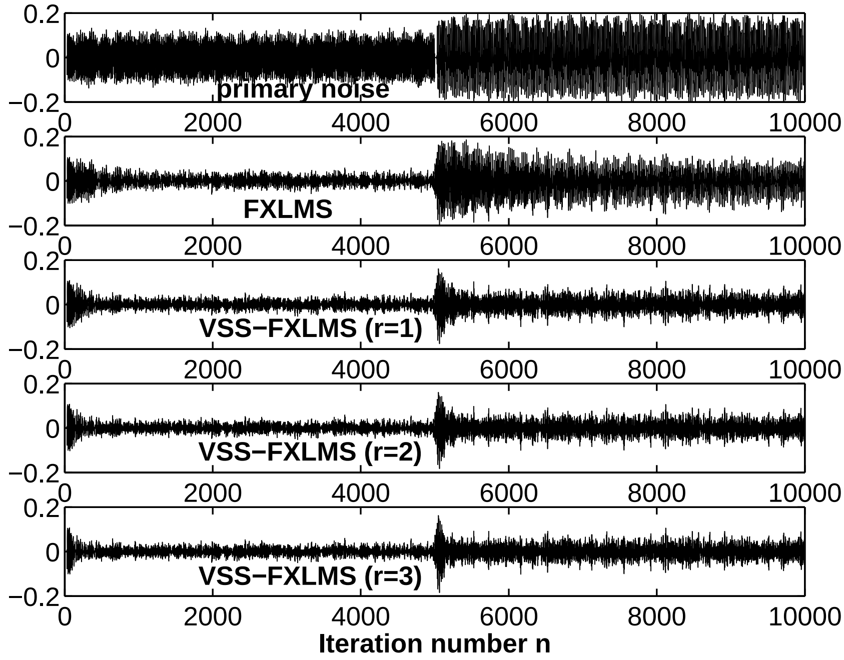

Fig. 5 Residual noises produced by the conventional FXLMS and our proposed VSS-FXLMS algorithm with different $r(1, 2, 3)$ in the presence of a real nonstationary primary noise.

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