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FLOW-AEROFOIL INTERACTION SOUND OF SUPERSONIC AEROFOILS

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1. INTRODUCTION

For high speed aerocraft using multistage propellers, one of the most noisy processes is the interaction of the moving propeller blades with the tip vortex flow shed from a previous propeller stage. When a blade cuts quickly through a vortex core flow, if the moving blade is not exactly parallel with the circular stream lines about the vortex core, as is the case in usual situations, the axial component of the vortex core flow is suddenly stopped by the blade, and the flow must adjust to the sudden change of the boundary constraints. This interruption of the streaming vortex core flow by the blade will inevitably involve compressive waves that propagate as sound to the far field. We believe that this is very likely to be an important mechanism by which high speed propellers generate noise, and the aim of this paper is to help understand better this mechanism. We choose to work with a canonical problem which locally models the interaction between a high speed propeller blade and a streaming vortex flow. We examine the sound generated from the interruption of a steady cylindrical flow by an aerofoil moving supersonically in the plane perpendicular to the steady flow. This is of course an enormously simplified model of the real situation, but we believe that it contains the essential features of the problem of propeller noise generation by flow-blade interactions, while being simple enough to be analysed exactly.

We examine the sound pressures produced by a semi-infinite aerofoil in section 2. The sound is generated in the form of a pressure pulse of finite duration. In directions other than the Mach angle, the pulse is switched on and off at zero amplitude, and has a maximum in the middle, which decreases due to spherical spreading inversely in proportion to the distance the pulse travels. The principal sound is launched in the Mach wave direction where the pressure pulse has sharp faces and constant amplitude; it reaches the far field without attenuation according to linear theory. For a real propeller system, this shock-like intense sound will of course undertake refractive interactions with the main inhomogeneous propulsion flow, and probably with other blades of the propeller system, before it reaches the distant observer, but those interactions may be regarded as the second stage of the complete flow-blade interaction sound problem which mainly concerns the propagation aspects, and may be considered separately from the generation problem.

Energetics of the flow-aerofoil interaction problem is examined in the subsequent sections, which reveals interesting results. In section 3, we examine the energy carried by the pressure pulse of the semi-infinite aerofoil. It will be shown that the sound energy is scattered from the steady jet flow, it being precisely equal to the loss of kinetic energy of the jet during the interruption process. For a semi-infinite aerofoil, the sound energy is independent of the supersonic aerofoil speed, and can be calculated by a very simple energy conservation argument. The scattered energy is also evaluated from the far field pressures, which confirms the result obtained through energy conservation arguments, and hence, demonstrates that there is no force at the supersonic leading edge which could significantly affect the acoustic radiation. The situation of finite chord aerofoils is considered in section 4. In this case, the radiated energy is a function of the aerofoil Mach number for small chord, but approaches the value for semi-infinite aerofoils when the chord is larger than the diameter of the cylindrical flow times $M+1$. For an aerofoil of fixed chord, the scattered energy is proportional to the inverse of the aerofoil Mach number at high supersonic speeds; the higher the speed, the quieter will be the sound of a supersonic propeller.

2. SOUND BY A SEMI-INFINITE AEROFOIL

We consider a semi-infinite aerofoil moving supersonically in its own plane perpendicular to a uniform cylindrical flow of radius a. The coordinates system is chosen in such a way that the aerofoil lies in the plane $x_3 = 0$, with its leading edge being parallel to the x_2 axis and advancing at speed cM, c being the constant sound speed and $M > 1$. The leading edge of the aerofoil coincides with the x_2 axis at time $t = 0$. The cylindrical flow has a uniform velocity u_0 in the negative x_3 direction, and its axis is chosen to be the x_3 axis.

The situation is illustrated in figure 1. We consider the sound generated linearly by the interruption of the flow by the moving aerofoil. The total flow field can then be regarded as the linear superposition of the cylindrical flow and an induced disturbance velocity field u_i. Due to the symmetrical geometry, it is sufficient to consider only the region $x_3 \geq 0$. The sound pressure can be conveniently calculated through the use of Kirchhoff's theorem [1], which relates the acoustic pressure at any observation position to the velocity distribution on the plane $x_3 = 0$, namely,

$$p(\mathbf{x},t) = \frac{\rho_0}{2\pi}\frac{\partial}{\partial t}\int_{y_\alpha}\left[\frac{u_3(\mathbf{y},\tau)}{|\mathbf{y}-\mathbf{x}|}\right]d^2y_\alpha, \tag{2.1}$$

where ρ_0 is the constant mean density, u_3 denotes the x_3-component of the induced velocity field u_i and the square brackets have the conventional retarded time implication, the quantity enclosed being evaluated at the retarded time $\tau = t - |\mathbf{y}-\mathbf{x}|/c$. The integrand of (2.1) is to be evaluated at $y_3 = 0$, where the distribution of u_3 can be specified as

$$u_3(\mathbf{y},\tau) = u_0 H(a^2 - y_\alpha^2)H(cMt - M|\mathbf{y}-\mathbf{x}| - y_1), \tag{2.2}$$

where H is the Heaviside step function. This specification follows from the fact that the induced velocity vanishes ahead of the leading edge of the aerofoil, because the leading edge moves faster than sound, while on the aerofoil surface, it also vanishes except in the region covered by the cylindrical flow, where it must be opposite to the flow velocity, to comply with the boundary condition that the total normal velocity on the aerofoil surface is zero. On substituting (2.2) into (2.1) and carrying out the partial derivative with respect to time t by noticing that the gradient of the Heaviside function is the Dirac delta function, we find that

$$p(\mathbf{x},t) = \frac{\rho_0 u_0 cM}{2\pi}\int_{y_\alpha}\frac{1}{|\mathbf{y}-\mathbf{x}|}H(a^2 - y_\alpha^2)\delta(cMt - M|\mathbf{y}-\mathbf{x}| - y_1)d^2y_\alpha. \tag{2.3}$$

The δ-function can be utilised to evaluate the y_2-integral. When this is done, we find that

$$p(\mathbf{x},t) = \frac{\rho_0 u_0 c}{2\pi}\int_{y_1}\frac{H(a^2 - y_1^2 - y_+^2) + H(a^2 - y_1^2 - y_-^2)}{\{(y_1 - cMt)^2 - M^2[(y_1 - x_1)^2 + x_3^2]\}^{1/2}}$$

$$\times\, H(cMt - y_1)H\left[(y_1 - cMt)^2 - M^2[(x_1 - y_1)^2 + x_3^2]\right]dy_1, \tag{2.4}$$

where $y_\pm$ are determined by solving the equation $cMt - M|\mathbf{y}-\mathbf{x}| - y_1 = 0$ for y_2, that is,

$$y_\pm = x_2 \pm \frac{1}{M}\left\{(y_1 - cMt)^2 - M^2[(y_1 - x_1)^2 + x_3^2]\right\}^{1/2}. \tag{2.5}$$

The determination of $y_\pm$ also results in the last two Heaviside functions in (2.4); when either of the arguments of the two is negative, $cMt - M|\mathbf{y}-\mathbf{x}| - y_1 = 0$ has no real solution for y_2, and hence the integral vanishes. Now, it can be recognised that (2.4) can be integrated to inverse sine functions (e.g. [2], formula 2.261),

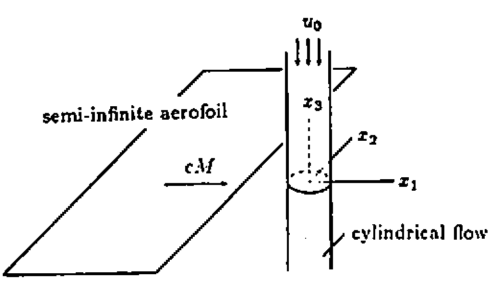

Figure 1. The geometry and the coordinate system of the model problem.

$$p(\mathbf{x},t) = \frac{\rho_0 u_0 c M}{2\pi\sqrt{M^2-1}} \arcsin \frac{(M^2-1)y_1 - M(Mx_1 - ct)}{M[(x_1 - cMt)^2 - (M^2-1)x_3^2]^{1/2}} \Bigg|_A^B , \qquad (2.6)$$

the integration bounds A and B being determined by the Heaviside functions in the integrand of (2.4). In doing so, quartic equations arising from the vanishing of the arguments of the Heaviside functions must be solved, which is algebraically complicated and tedious, except in the case of $x_2 = 0$ which we will examine in the next section. It is straightforward, however, to solve these equations numerically. The sound pressure can then be obtained according to (2.6).

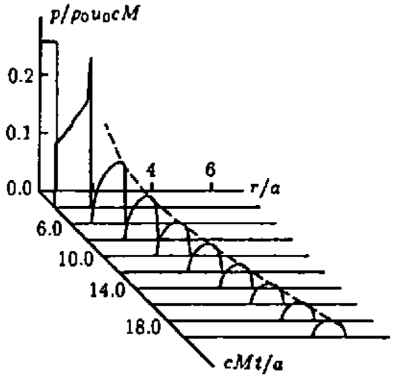

Figure 2. Sound pressures calculated according to (2.6) with $M = 4.0$ in the direction $\phi = \theta = \pi/4$. The $1/r$ decrease is illustrated by the dash curve.

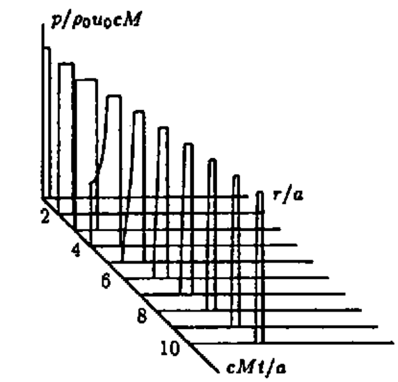

Figure 3. A constant pressure pulse is launched in the Mach wave direction $\phi = 0$ and $\theta = \arcsin(1/M)$, which has a magnitude of $1/\sqrt{M^2-1}$.

Plotted in figure 2 are the sound pressures as a function of the radial position r for $\phi = \theta = \pi/4$, where (r, ϕ, θ) is the spherical coordinates defined by $x_1 = r\sin\theta\cos\phi$, $x_2 = r\sin\theta\sin\phi$ and $x_3 = r\cos\theta$. This figure is representative of any directions other than that along the Mach angle. Apparently, the sound is generated in the form of a pressure pulse that is switched on and off at zero amplitude. The maximum amplitude occurs in the middle of the pulse. As time increases, the wave form propagates away at constant speed c with its maximum amplitude decresing according to $1/r$, or, equivalently $1/ct$, obviously due to spherical spreading, which is illustrated in figure 2 by the dash curve. The pressure vs radius curves are all smooth and rounded in the far field, but have sharp faces in the near field (the curves corresponding to $cMt/a = 2.0$ and 4.0) when the leading edge of the aerofoil is within the cylindrical flow, because the pressure perturbations in this case jump from zero to a constant value across the edge of the Mach cone from the leading edge. As time t increases, the initial rectangular pressure form travels to the far field with its height decreasing due to spherical spreading. That decrease starts at the rear face of the pressure pulse, so that, when $cMt/a = 4.0$, only a very narrow part close to the front face of the initial rectangular wave form still remains its initial height, while most of the pulse has greatly decreased in amplitude. This gives the peak (of finite height) in the second curve at $cMt/a = 4.0$ in figure 2.

In the Mach wave direction dead ahead of the moving aerofoil, that is , in the direction of $\phi = 0$ and $\sin\theta = 1/M$, the amplitude of the pressure pulse is unaffected by spherical spreading . This indicates that a pressure pulse of constant amplitude is launched in this direction. This situation is shown in figure 3. It is clear that a sharp fronted pulse is generated by the impact on the flow of the moving aerofoil. This pressure pulse propagates along the Mach wave direction at speed c without attenuation. The rear face of the wave form also becomes sharp as it travels away. An intense bang will be heard by distant observers in this Mach wave direction.

3. THE SCATTERED ENERGY

The sound energy scattered by the moving aerofoil can be found by considering the energy loss of the total flow during the interruption process, that is, by calculating the difference between the energy contained in the flow before the aerofoil cuts the cylindrical flow and that remaining in the field after the interruption process. That energy difference must have escaped to infinity as sound since the dragless aerofoil moves in its own plane and does *no* work. It is evident that the leading edge of the supersonically moving aerofoil does not experience any suction force which could significantly affect the acoustic radiation, as it might do in subsonic situations [3], because the supersonic leading edge cannot be affected by the disturbances generated by the leading edge itself. It is also evident that no energy is radiated from the aerofoil surface because the energy flux on the plane $x_3 = 0$, in which the aerofoil lies, is identically zero, due to the fact that the total normal velocity on the aerofoil surface ($x_1 < cMt$) always vanishes and the pressure perturbations are always zero ahead of the leading edge ($x_1 > cMt$) that advances at supersonic speed. Hence the total energy (the acoustic and the kinetic energy) in the field is conserved during the whole process of interruption of the steady flow by the semi-infinite aerofoil. The aerofoil generates sound by scattering the jet's energy into sound, the strength of which can be evaluated by energy conservation arguments.

Suppose that the main flow, of velocity $v_i(\mathbf{x})$ with $\partial v_i/\partial x_i = 0$, is interrupted by the aerofoil, producing a disturbance velocity potential ϕ, so that the total velocity of the interrupted flow is $v_i + \partial\phi/\partial x_i$. In our particular case where the jet flow is perpendicular to the moving aerofoil, v_i only has one component in the negative x_3 direction (noting that the flow does not have a uniform velocity component in the x_1 direction). At time $t \to -\infty$ when the aerofoil is still far away from the flow v_i, there is no disturbance in the field and the total energy contained in the flow is given by

$$E_{-\infty} = \int \frac{1}{2}\rho_0 \left[v_i(\mathbf{x})\right]^2 d^3\mathbf{x}, \tag{3.1}$$

where the volume integral is over the entire space. As time $t \to +\infty$, the leading edge of the semi-infinite aerofoil will move far downstream. The flow is then effectively bounded by an infinite plane boundary at $x_3 = 0$; the interrupted flow will eventually settle down to a steady state with the induced velocity potential determined by the Laplace equation $\nabla^2\phi = 0$ and the boundary condition that the total normal velocity on the aerofoil surface must vanish.

$$\frac{\partial\phi}{\partial x_3} = -v_3 \quad \text{on} \quad x_3 = 0 \tag{3.2}$$

then imposes the solution for ϕ [5]

$$\phi = \frac{1}{2\pi}\int_{y_\alpha} \frac{v_3(\mathbf{y})}{|\mathbf{y} - \mathbf{x}|} d^2 y_\alpha. \tag{3.3}$$

This is the near field motion which does not die out with time, so that the total energy remaining in the field at time $t \to +\infty$ is

$$E_{+\infty} = \int \frac{1}{2}\rho_0 \left[v_i + \frac{\partial \phi}{\partial x_i}\right]^2 d^3\mathbf{x}, \tag{3.4}$$

By making use of the relations $\partial v_i/\partial x_i = 0$ and $\nabla^2\phi = 0$, this can be rewritten as

$$E_{+\infty} = E_{-\infty} + \frac{1}{2}\rho_0 \int \frac{\partial}{\partial x_i}\left(2\phi v_i + \phi\frac{\partial \phi}{\partial x_i}\right) d^3\mathbf{x}, \tag{3.5}$$

where $E_{-\infty}$ is the energy in the flow at time $t \to -\infty$ and is given by (3.1). The scattered acoustic energy E_a can now be found by calculating the difference between $E_{-\infty}$ and $E_{+\infty}$.

$$E_a = E_{-\infty} - E_{+\infty} = \rho_0 \int_{x_\alpha} \left(2\phi v_3 + \phi\frac{\partial \phi}{\partial x_3}\right) d^2x_\alpha, \tag{3.6}$$

where the last step follows from applying the divergence theorem to the volume integral in (3.5). In doing so, we have utilized the symmetrical geometry of the problem and performed the volume integral only in the upper half space with the result doubled. On the surface of the aerofoil, we have the boundary condition (3.2) for the normal velocity and (3.3) for the velocity potential, so that for any initially steady velocity field v_i, the energy scattered into sound by a semi-infinite aerofoil is

$$E_a = \frac{1}{2\pi}\rho_0 \int_{x_\alpha}\int_{y_\alpha} \frac{v_3(\mathbf{x})v_3(\mathbf{y})}{|\mathbf{y}-\mathbf{x}|} d^2x_\alpha d^2y_\alpha. \tag{3.7}$$

This result is derived without any particular specification of either the value of the supersonic speed of the aerofoil or the particular distribution of the basic flow field v_i, provided that it is steady with finite dimensions in the plane in which the aerofoil moves. It can be seen from (3.7) that the scattered sound energy is independent of the Mach number of the supersonic aerofoil. For our uniform cylindrical flow, we have

$$v_3(\mathbf{x}) = -u_0 H(a^2 - x_\alpha^2). \tag{3.8}$$

Hence, (3.7) becomes

$$E_a = \frac{1}{2\pi}\rho_0 u_0^2 \int_{x_\alpha}\int_{y_\alpha} \frac{H(a^2 - x_\alpha^2)H(a^2 - y_\alpha^2)}{|y_\alpha - x_\alpha|} d^2y_\alpha d^2x_\alpha. \tag{3.9}$$

The x_α-integral can be carried out immediately (e.g. [5]),

$$\int_{x_\alpha} \frac{H(a^2 - x_\alpha^2)}{|y_\alpha - x_\alpha|} d^2x_\alpha = 4a\mathbf{E}\left(\frac{|y_\alpha|}{a}\right), \tag{3.10}$$

where $\mathbf{E}(z)$ is the complete elliptic integral of the second kind. Thus, (3.9) yields

$$E_a = \frac{2a\rho_0 u_0^2}{\pi} \int_{y_\alpha} \mathbf{E}\left(\frac{|y_\alpha|}{a}\right) H(a^2 - y_\alpha^2) d^2y_\alpha = \frac{8}{3}\rho_0 u_0^2 a^3, \tag{3.11}$$

where the last step is derived by changing the integration variables to polar coordinates and using (8.112) and (6.132) of [2]. This is the sound energy scattered by the semi-infinite aerofoil, which is only related to the parameters of the cylindrical jet flow and is independent of the supersonic aerofoil speed. This result, derived through arguments of energy conservation, can also be obtained directly from the far field acoustic motions, which we do next. We start with the far field acoustic pressure perturbations that can be found from (2.3) by approximating $1/|\mathbf{y}-\mathbf{x}|$ by $1/|\mathbf{x}|$ and $|\mathbf{y}-\mathbf{x}|$ by $|\mathbf{x}| - y_\alpha \hat{x}_\alpha$. Thus we have

$$p \sim \frac{\rho_0 u_0 cM}{2\pi|\mathbf{x}|} \int_{y_\alpha} H(a^2 - y_\alpha^2)\delta(cMt - y_1 - M|\mathbf{x}| + M\hat{x}_\alpha y_\alpha) d^2y_\alpha,$$

$$= \frac{\rho_0 u_0 c M}{(2\pi)^2|\mathbf{x}|}\int_{y_\alpha}\int_k H(a^2 - y_\alpha^2)e^{ik(cMt - y_1 - M|\mathbf{x}| + M\hat{x}_\alpha y_\alpha)}d^2y_\alpha dk.$$

The total acoustic energy can be found by integrating the square of this result, divided by $\rho_0 c$, which gives the far field energy flux in the radial direction, over a spherical surface of large radius $|\mathbf{x}|$ from time $t \to -\infty$ to $t \to +\infty$. This yields the result for the sound energy E_a,

$$E_a = \frac{2\rho_0 u_0^2 M}{(2\pi)^3}\int H(a^2 - y_\alpha^2)H(a^2 - z_\alpha^2)e^{ik[M\hat{x}_\alpha(y_\alpha - z_\alpha) - (y_1 - z_1)]}\sin\theta d\theta d\phi dk d^2y_\alpha d^2z_\alpha, \qquad (3.12)$$

where the θ-integral is from 0 to $\pi/2$ and that with respect to ϕ from 0 to 2π. Other integrals are all performed from $-\infty$ to $+\infty$. Now we can carry out the ϕ-integral with the result expressed in terms of a Bessel function which can then be integrated with respect to θ, according to (6.554) of [2], to give

$$\int_0^{2\pi}\int_0^{\pi/2} e^{ikM\hat{x}_\alpha(y_\alpha - z_\alpha)}\sin\theta d\theta d\phi = 2\pi\frac{\sin(kM|y_\alpha - z_\alpha|)}{kM|y_\alpha - z_\alpha|}.$$

Substituting this into (3.12), and performing the k-integral, we find that

$$E_a = \frac{\rho_0 u_0^2}{4\pi}\int_{y_\alpha}\int_{z_\alpha}\left[\operatorname{sgn}\left(|y_\alpha - z_\alpha| + \frac{y_1 - z_1}{M}\right) + \operatorname{sgn}\left(|y_\alpha - z_\alpha| - \frac{y_1 - z_1}{M}\right)\right]$$
$$\times\frac{H(a^2 - y_\alpha^2)H(a^2 - z_\alpha^2)}{|y_\alpha - z_\alpha|}d^2y_\alpha d^2z_\alpha.$$

Since the Mach number M is larger than one, the arguments of both the two sign functions in the integrand are always positive. Hence the quantity enclosed in the square brackets is identically equal to 2. This result is then seen to be exactly the same as (3.9), and hence (3.11), the result derived through energy conservation arguments. The agreement of the two calculations also gives evidence that the supersonic leading edge is indeed dragless, because the far field calculation does not involve any assumption at the leading edge, but gives the same result as that of the energy conservation arguments which presume a dragless leading edge.

4. ENERGY SCATTERED BY A FINITE CHORD AEROFOIL

In practice, aerofoils have finite chord and it is obviously of interest to see whether the finiteness of the aerofoils can significantly affect the generated sound. We consider an aerofoil of finite chord $2b$ that moves supersonically in the plane $x_3 = 0$. Since the disturbances caused by the trailing edge of the supersonic aerofoil cannot affect the field ahead of it, the pressure fluctuations on the finite aerofoil surface are precisely the same as those on a semi-infinite aerofoil which is given by (2.3) with $x_3 = 0$. Because the pressure fluctuations vanish elsewhere in the plane $x_3 = 0$ due to symmetry, (2.3) can be utilised to find the pressure perturbations in the field according to Kirchhoff's theorem, and hence, the acoustic energy radiated to infinity. The total energy can also be calculated through the evaluation of the energy across the plane control surface just above the coordinate plane $x_3 = 0$. In this event, the velocity distribution on the aerofoil has the same specification as that in the semi-infinite case, both being determined according to the vanishing normal velocity boundary condition. Behind the trailing edge, the velocity distribution is unknown, but this does not affect the energy calculation, because the pressure fluctuations there always vanish due to symmetry, and so too does the energy flux. Hence the energy flux in the plane $x_3 = 0$ for this finite chord situation is given by

$$u_0[H(cMt - x_1) - H(cMt - x_1 - 2b)]$$

times the pressure (2.3). Integrating the result over time t and space x_α, we derive an expression

$$E_b = \frac{\rho_0 u_0^2}{\pi}\int_{y_\alpha}\int_{x_\alpha}\frac{H(a^2 - y_\alpha^2)H(a^2 - x_\alpha^2)}{|y_\alpha - x_\alpha|}H[2b - (y_1 - x_1) - M|y_\alpha - x_\alpha|]d^2y_\alpha d^2x_\alpha, \qquad (4.1)$$

where we have multiplied the result by two to take account of the energy into the lower half space, so that E_b is the total energy produced by the aerofoil. The integrals in (4.1) can be evaluated by changing the integration variables x_o and y_o into polar coordinates, which transfer (4.1) to

$$E_b = \frac{\rho_0 u_0^2}{2\pi} \int H[a^2 - k^2 - \lambda^2 - 2k\lambda\cos(\alpha - \beta)] H[2b - k(\cos\alpha + M)] \lambda d\lambda dk d\alpha d\beta, \tag{4.2}$$

where both the α-integral and the β-integral are performed from 0 to 2π, the λ-integration is from 0 to a and that with respect to k along the positive k-axis. In (4.2), we can replace $\cos(\alpha - \beta)$ in the first Heaviside function by $\cos\beta$, because the β-integral is over a complete period. Thus, the α-integral and the λ-integral can be evaluated separately, by utilizing respectively the two Heaviside functions. The result can then be further calculated as

$$E_b = \begin{cases} \frac{16}{3}\rho_0 u_0^2 a^3 & \text{when} \quad \frac{a}{b} < \frac{1}{M+1} < \frac{1}{M-1}, \\ \frac{16}{3}\rho_0 u_0^2 a^3 \left[1 + \frac{b}{2\pi a} f(1)\right] & \text{when} \quad \frac{1}{M+1} < \frac{a}{b} < \frac{1}{M-1}, \\ \frac{16}{3}\rho_0 u_0^2 a^3 \left[1 + \frac{b}{2\pi a} f\left(\frac{b}{a(M-1)}\right)\right] & \text{when} \quad \frac{1}{M+1} < \frac{1}{M-1} < \frac{a}{b}, \end{cases} \tag{4.3}$$

where $f(z)$ is a complicated combination of elementary functions and elliptic functions, but it can also be expressed by a simple integral as

$$f(z) = \int^z \frac{3\eta \arccos\eta - (2 + \eta^2)\sqrt{1 - \eta^2}}{\eta\sqrt{2M\eta b/a - b^2/a^2 - (M^2 - 1)\eta^2}} d\eta,$$

with the lower limit of integration given by $b/a(M+1)$.

These results are shown in figure 4, where the energy produced by the finite chord aerofoil is plotted as a function of the aerofoil Mach number M. Apparently, the upper bound of the radiated energy is that of a semi-infinite aerofoil. This maximum energy is achieved, not as a limiting process as the aerofoil chord tends to infinity, but at a condition $b > a(M+1)$, as indicated by (4.3). For aerofoils with chord larger than $2a(M+1)$, the radiated energy is independent on the aerofoil Mach number just as a semi-infinite aerofoil. As analysed in section 4, the supersonic trailing edge in this case of large chord aerofoil behaves in the same way as that of the semi-infinite plane, and has no influence on the flow field.

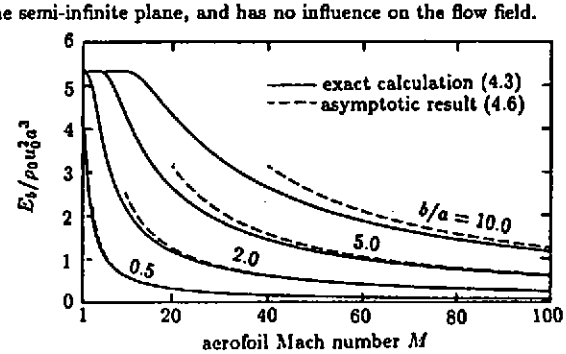

Figure 4. The scattered energy by a finite chord aerofoil.

For a fixed aerofoil chord, the radiated energy decreases as the Mach number M increases, indicating that the faster the supersonic aerofoil moves, the quieter the sound field will be. The decrease of E_b can be shown to be $1/M$ at high values of M. It is straightforward to demonstrate this by evaluating (4.1) in the limiting case of $M \gg 1$. The x_o-integral can be performed, in this event, as

$$\int_0^{2\pi} [S_1(\alpha) H(S_0 - S_1) + S_0(\alpha) H(S_1 - S_0)] d\alpha, \tag{4.4}$$

where $S_1(\alpha) = \sqrt{a^2 - y_a^2\sin^2(\alpha-\beta)} - |y_a|\cos(\alpha-\beta)$, $S_0(\alpha) = 2b/(M+\cos\alpha)$ and $\beta = \arctan(y_2/y_1)$. Apparently, $H(S_0 - S_1)$ vanishes and $H(S_1 - S_0) \to 1$ in the case of $M \gg 1$. Hence, (4.4) reduces to

$$\int_0^{2\pi} \frac{2b}{M+\cos\alpha} d\alpha = \frac{4\pi b}{\sqrt{M^2-1}}. \tag{4.5}$$

Equations (4.5) and (4.1) then yield that, for $M \gg 1$,

$$E_b \sim \frac{2\pi\rho_0 u_0^2 a^2 b}{M}. \tag{4.6}$$

This result clearly reveals a $1/M$ dependence of the radiated energy; it decreases as the supersonic aerofoil speed increases, mainly because of the reducing duration of the pressure pulse at high supersonic speed. The asymptotic solution (4.6) is compared, in figure 4, with the exact calculations of (4.3). The result (4.6) also predicts a linear relation between the energy and the chord of the aerofoil. The linear increase of sound with the dimension of the aerofoil is true only for the case of high supersonic speed and moderate chord; for fixed M and very large b, (4.4) reduces to (3.10) instead of (4.5), because $H(S_0 - S_1)$ would then be equal to unity and $H(S_1 - S_0)$ vanishes, and the energy is given in this case by the first line of (4.3).

5. CONCLUSIONS

We have examined a flow-aerofoil interaction problem. The sound generated by a semi-infinite plane with a leading edge moving supersonically through and interrupting a uniform cylindrical flow has been found exactly. The generated sound takes the form of a pressure pulse of finite duration. In directions other than the Mach wave direction, the pressure pulse is switched on and off at zero amplitude, and has a maximum in the middle, which decreases, due to spherical spreading, inversely in proportion to the distance that the pulse travels. Along the Mach angle, it has been found that the pulse has sharp faces (both front and rear) and constant amplitude; it reaches distant observers without attenuation according to liner theory. It is the noisiest sound from the flow-aerofoil interaction and is, we believe, likely to be one of the most important aspects concerning the noise of aircraft using supersonic propellers.

The energy radiated to infinity has been calculated explicitly. For a semi-infinite aerofoil, the sound is scattered from the uniform jet flow; the loss of kinetic energy of the jet flow during the whole interruption process has been shown to be precisely equal to the energy radiated as sound to infinity. The scattered acoustic energy has been shown to be independent of the Mach number at which the supersonic semi-infinite plane moves. By calculating the sound energy radiated to infinity and considering the conservation of energy, it has been demonstrated that the supersonically moving aerofoil does not experience any force whose work might affect the sound generation process. For aerofoils of finite chord, it has been shown that the radiated energy is a function of the aerofoil Mach number for small chord, but approaches the value for semi-infinite aerofoils when the chord is larger than the diameter of the cylindrical flow times $M+1$. At high supersonic speeds, finite chord aerofoils radiate energy that decreases inversely as the Mach number of the finite chord aerofoil increases; the higher the speed, the quieter will be the sound generated as a result of flow scattering by a finite chord supersonic aerofoil.

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