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VIBRATION ANALYSIS OF DOUBLE-DECK SANDWICH PLATES WITH LATTICE CORES

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Vibration of the double-deck sandwich plate with pyramidal lattice core is investigated in the present paper. The structure consists of three thin panels and two lattice cores. The lattice truss core is modeled by a continuous material. The equation of motion of the double-deck lattice sandwich plate is established by Hamilton's principle in which the displacements are expressed with a simple method, and the natural frequencies of the structure are calculated. The effects of structural parameters on the vibration characteristics of the double-deck lattice sandwich plate are analyzed and discussed. The present method will be useful for the vibration analysis and design of the sandwich plates with lattice truss core.

Keywords: Sandwich plate; lattice truss core; free vibration; Hamilton's principle; natural frequency

1. Introduction

Sandwich structures which comprise thin face-sheets and thick cores have been used widely in the routine life. The face-sheets can be different in thickness and material, and they mainly support the bending loads and in-plane loads. The cores can be honeycomb, corrugated, truss core or foam, and they primarily support the transverse shear loads.

Some theoretical and experimental studies on the vibration analysis of the sandwich structures have been conducted. Reddy and Kuppusamy [1] analyzed the free vibration of the orthotropic anisotropic laminate plate, and Reddy and Phan [2] studied the stability and vibration of orthotropic isotropic and anisotropic laminate plate later. Wicks and Hutchinson [3] investigated a tetrahedral truss core and either solid or triangulated face-sheets. They found the weights of the optimized truss core plates for a given bending and shear load were similar to the honeycomb-core sandwich plates. Hwu et al. [4] got the closed-form solution for the free vibration problems of sandwich beam and derived an orthogonality relation consisting the effects of rotary inertia and shear deformation.

Lok and Cheng [5] derived the equivalent stiffness of an orthotropic sandwich panel and obtained good agreement of responses between analytical solutions and the FEM results. Liu et al. [6] raised an equivalent single layered finite element computational model to predict the structural behavior of truss core sandwich panels. Xu and Qiu [7] investigated the natural frequencies of composite sandwich beams with lattice truss core by combining the Bernoulli-Euler beam theory and Timoshenko beam theory. Chen et al. [8] studied the vibration and buckling of truss core sandwich plates on an elastic foundation subjected to biaxial in-plane loads, and they found that the lowest buckling loads and modes were dependent on the foundation stiffness as well as bending and transverse shear stiffness of the plate.

There have been many investigations on the vibrations in different forms of sandwich structures, however, it is still essential to explore simple and effective methods to study the vibration problems of the complex double-deck sandwich plates with lattice cores. In the present work, a convenient and simple approach is developed to research the free vibration of double-deck sandwich plate with pyramidal truss core. The lattice truss core is idealized as homogeneous material in which the transverse and in-plane shear deformation is considered. For the thin face-sheets, the bending deformation is considered, and the effect of the transverse shear deformation is neglected. The displacements are expressed by a simple method and the natural frequencies are conveniently calculated. Then the effects of the structural parameters on the natural frequencies of the double-deck lattice sandwich plate are analyzed and discussed.

2. Formulations of theoretical model

Fig. 1 shows the double-deck sandwich plate with lattice truss core whose unit cell of the pyramidal truss core consisting of four struts is displayed in Fig. 2. The upper and lower face-sheets share the same thickness h_f, the two truss cores share the same height h_c, and the thickness of the middle thin panel is h_m. The unit cell length is d and the inclination angle of the truss is denoted by θ, and all struts have the same length L_c and cross-sectional area A_c. The total sandwich structure share the same mother material whose density is ρ. The equivalent density of the truss core can be expressed as $\rho_c = \frac{4L_c A_c \rho}{h_c d^2}$, and the transverse shear modulus of the truss core is given by Ref. [8]: $G_{xzc} = G_{yzc} = \frac{E_s}{8}\bar{\rho}\sin^2 2\theta$, in which E_s is the Young's modulus of the mother material, and $\bar{\rho} = \frac{\rho_c}{\rho}$ is the relative density of the core.

Fig. 3 shows deformation pattern of the micro-unit with length dx and width dy. The thin face-sheets will rotate by an angle $\partial w/\partial x$ and $\partial w/\partial y$ around the y and x axes. It is assumed that the truss core will rotate by an angle $\lambda\partial w/\partial x$ and $\lambda\partial w/\partial y$ around the y and x axes, where λ is an undetermined coefficient.

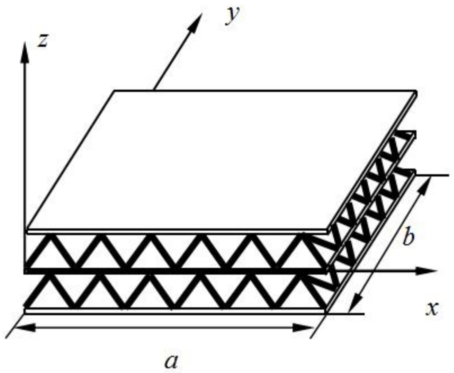

Fig. 1 The schematic diagram of the sandwich plate.

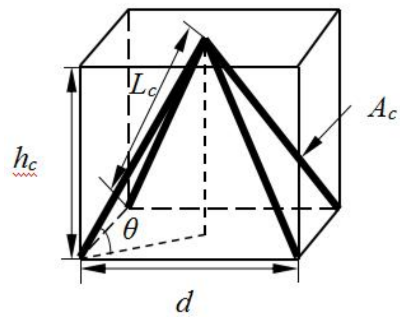

Fig. 2 Pyramidal lattice core.

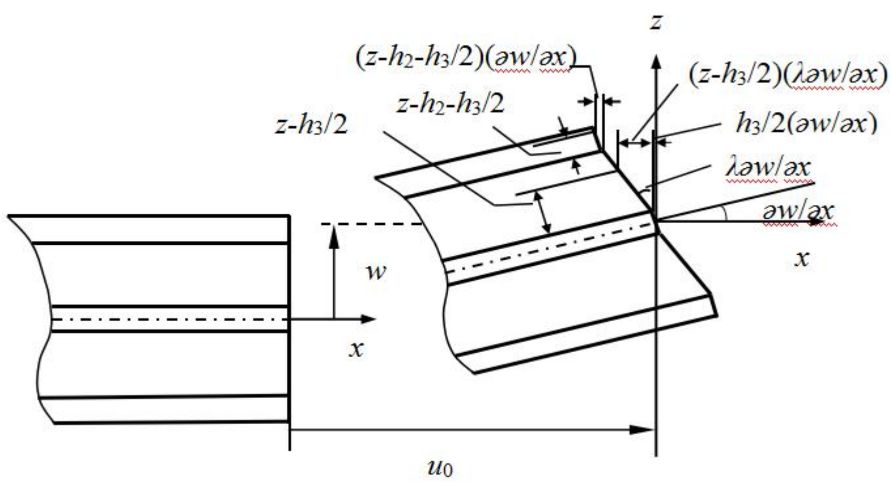

Fig. 3 Deformation diagram of the micro-unit.

The displacements in the *x*-direction and *y*-direction for different layers of the sandwich plate are given as follows:

$$u_1 = (-z + h_2 - \lambda h_2)\frac{\partial w}{\partial x},\ v_1 = (-z + h_2 - \lambda h_2)\frac{\partial w}{\partial y}, \tag{1a}$$

$$u_2 = (-\lambda z - \frac{h_3}{2} + \lambda\frac{h_3}{2})\frac{\partial w}{\partial x},\ v_2 = (-\lambda z - \frac{h_3}{2} + \lambda\frac{h_3}{2})\frac{\partial w}{\partial y}, \tag{1b}$$

$$u_3 = -z\frac{\partial w}{\partial x},\ v_c = -z\frac{\partial w}{\partial y}, \tag{1c}$$

$$u_4 = (-\lambda z + \frac{h_3}{2} - \lambda\frac{h_3}{2})\frac{\partial w}{\partial x},\ v_4 = (-\lambda z + \frac{h_3}{2} - \lambda\frac{h_3}{2})\frac{\partial w}{\partial y}, \tag{1d}$$

$$u_5 = (-z - h_4 + \lambda h_4)\frac{\partial w}{\partial x},\ v_5 = (-z - h_4 + \lambda h_4)\frac{\partial w}{\partial y}, \tag{1e}$$

where the subscripts 1, 3, 5 represent the upper, middle and lower face-sheets, and 2, 4 represent the two truss cores. The strains in the different layers of the sandwich plate can be obtained as follows:

$$\varepsilon_{xx1} = (-z + h_2 - \lambda h_2)\frac{\partial^2 w}{\partial x^2},\ \varepsilon_{yy1} = (-z + h_2 - \lambda h_2)\frac{\partial^2 w}{\partial y^2},$$

$$\gamma_{xy1} = 2(-z + h_2 - \lambda h_2)\frac{\partial^2 w}{\partial x \partial y}, \tag{2a}$$

$$\gamma_{yz2} = (1-\lambda)\frac{\partial w}{\partial y},\ \gamma_{xz2} = (1-\lambda)\frac{\partial w}{\partial x}, \tag{2b}$$

$$\varepsilon_{xx3} = -z\frac{\partial^2 w}{\partial x^2},\ \varepsilon_{yy3} = -z\frac{\partial^2 w}{\partial y^2},\ \gamma_{xy3} = -2z\frac{\partial^2 w}{\partial x \partial y}, \tag{2c}$$

$$\gamma_{yz4} = (1-\lambda)\frac{\partial w}{\partial y},\ \gamma_{xz4} = (1-\lambda)\frac{\partial w}{\partial x}, \tag{2d}$$

$$\varepsilon_{xx5} = (-z - h_4 + \lambda h_4)\frac{\partial^2 w}{\partial x^2},\ \varepsilon_{yy5} = (-z - h_4 + \lambda h_4)\frac{\partial^2 w}{\partial y^2},$$

$$\gamma_{xy1} = 2(-z - h_4 + \lambda h_4)\frac{\partial^2 w}{\partial x \partial y}, \tag{2e}$$

where ε and γ are the normal and shear strains.

The stresses in the different layers of the sandwich plate can also be obtained by the Hooke's law as follows:

$$\sigma_{xx1} = \frac{E}{1-\upsilon^2}(-z + h_2 - \lambda h_2)(\frac{\partial^2 w}{\partial x^2} + \upsilon\frac{\partial^2 w}{\partial y^2}),\ \sigma_{yy1} = \frac{E}{1-\upsilon^2}(-z + h_2 - \lambda h_2)(\frac{\partial^2 w}{\partial y^2} + \upsilon\frac{\partial^2 w}{\partial x^2}),$$

$$\tau_{xy1} = \frac{E}{1+\upsilon}(-z + h_2 - \lambda h_2)\frac{\partial^2 w}{\partial x \partial y}, \tag{3a}$$

$$\tau_{yz2} = G_{yz}(1-\lambda)\frac{\partial w}{\partial y},\ \tau_{xz2} = G_{xz}(1-\lambda)\frac{\partial w}{\partial x}, \tag{3b}$$

$$\sigma_{xx3} = -\frac{Ez}{1-\upsilon^2}(\frac{\partial^2 w}{\partial x^2} + \upsilon\frac{\partial^2 w}{\partial y^2}),\ \sigma_{xx3} = -\frac{Ez}{1-\upsilon^2}(\frac{\partial^2 w}{\partial y^2} + \upsilon\frac{\partial^2 w}{\partial x^2}),\ \tau_{xy3} = -\frac{Ez}{1+\upsilon}\frac{\partial^2 w}{\partial x \partial y}, \tag{3c}$$

$$\tau_{yz4} = G_{yz}(1-\lambda)\frac{\partial w}{\partial y},\ \tau_{xz4} = G_{xz}(1-\lambda)\frac{\partial w}{\partial x}, \tag{3d}$$

$$\sigma_{xx5} = \frac{E}{1-\upsilon^2}(-z-h_4+\lambda h_4)(\frac{\partial^2 w}{\partial x^2} + \upsilon\frac{\partial^2 w}{\partial y^2}),\ \sigma_{yy1} = \frac{E}{1-\upsilon^2}(-z-h_4+\lambda h_4)(\frac{\partial^2 w}{\partial y^2} + \upsilon\frac{\partial^2 w}{\partial x^2}),$$

$$\tau_{xy5} = \frac{E}{1+\upsilon}(-z-h_4+\lambda h_4)\frac{\partial^2 w}{\partial x \partial y}, \tag{3e}$$

where σ and τ are the normal and shear stresses.

The equations of motion of the sandwich plate can be established by Hamilton's principle

$$\delta\int_{t_0}^{t_1}(T-U)dt = 0, \tag{4}$$

where T and U are the kinetic and strain energies of the structure, δ is the variational operator, and t is the time.

Substituting Eq. (1), (2) and (3) into Eq. (4), and performing the variation operation in terms of w, the equation of motion of the lattice sandwich plate can be obtained as

$$\frac{E}{1-\upsilon^2}A\nabla^2\nabla^2 w - (\lambda-1)^2(h_2+h_4)(G_{yz}\frac{\partial^2 w}{\partial y^2} + G_{xz}\frac{\partial^2 w}{\partial x^2}) = B(\frac{\partial^2 \ddot{w}}{\partial x^2} + \frac{\partial^2 \ddot{w}}{\partial y^2}) - [\rho(h_1+h_3+h_5) + (\rho_2+\rho_4)]\ddot{w} \tag{5}$$

where A, B are given in Appendix.

For simply supported sandwich plates, the boundary conditions can be written as

$$w = 0, M_x = 0,\ \text{for } x = 0 \text{ and } a,$$
$$w = 0, M_y = 0,\ \text{for } y = 0 \text{ and } b, \tag{6}$$

where a and b are length and width of the sandwich plate. The transverse deflection function can be expressed as

$$w(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\frac{\sin m\pi x}{a}\frac{\sin n\pi y}{b}\sin(\omega_{mn}t + \varphi_0), \tag{7}$$

where φ_0 is the initial phase angle, and A_{mn} and ω_{mn} are the amplitude and angular frequency of the transverse displacement.

Substituting Eq. (7) into Eq. (5), the frequency equation for simply supported sandwich plates can be obtained as

$$\frac{E}{1-\upsilon^2}A[(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2]^2 + (\lambda-1)^2(h_2+h_4)[G_{yz}(\frac{n\pi}{b})^2 + G_{xz}(\frac{m\pi}{a})^2] = B[(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2]\omega_{mn}^2 + [\rho(h_1+h_3$$
$$+h_5) + \rho_c(h_2+h_4)]\omega_{mn}^2 \tag{8}$$

According to the force balance equation, one has

$$\frac{\partial Q_{xz}}{\partial x} + \frac{\partial Q_{yz}}{\partial y} = [\rho(h_1+h_3+h_5) + \rho_c(h_2+h_4)]\frac{\partial^2 w}{\partial t^2}, \tag{9}$$

in which

$$Q_{yz} = (1-\lambda)(h_2+h_4)G_{yzc}\frac{\partial w}{\partial y},\ Q_{xz} = (1-\lambda)(h_2+h_4)G_{xzc}\frac{\partial w}{\partial x}. \tag{10}$$

Substituting Eqs. (7) and (10) into Eq. (9), the expression for the coefficient λ can be expressed as

$$\lambda = 1 - \frac{[\rho(h_1 + h_3 + h_5) + \rho_c(h_2 + h_4)]\omega_{mn}^2}{(h_2 + h_4)[G_{xzc}(\frac{m\pi}{a})^2 + G_{yzc}(\frac{n\pi}{b})^2]}. \tag{11}$$

Now, substituting Eq. (11) into Eq. (8), the natural frequencies of the sandwich plate ω_{mn} can be easily obtained.

3. Results and analysis

The dimensions and material parameters of the sandwich plate are given as follows:
E = 210GPa, ν = 0.3, ρ = 7800kg/m^3, a = 1.06m, b = 1.06m, h_c = 0.03m, d = 0.0424m, h_f = 0.0025m, h_m=0.0025m, and $A_c = 9\times10^{-6}$m. And the results are shown in Table 1.

Table 1 Comparison of the natural frequencies (Hz) of a simply supported sandwich plate.

Modes (m, n)	(1,1)	(1,2), (2,1)	(2,2)	(1,3), (3,1)	(2,3), (3,2)
Frequency (Hz)	526	831	1051	1175	1340

Then the effect of the thickness of the face-sheets, the height of the truss cores, and the cross-sectional size of the struts on the vibration characteristics of the double-deck sandwich plate are discussed. The cross-section of the struts is assumed to be square, and its length of side is expressed by t_c. Other parameters are mentioned before.

Fig. 4.1 shows the effect of the thickness of the upper and lower face-sheets on the first four natural frequencies of the lattice sandwich plate. It can be found that all the first four natural frequencies decrease with the thickness of the face-sheets increasing. The effect of the thickness of the middle face-sheet on the first four natural frequencies of the lattice sandwich plate are similar in Fig. 4.2, which shows that as the the thickness of the face-sheet increasing, all the natural frequencies decrease. Fig. 4.3 shows the effect of the cross-sectional size of the struts on the first four natural frequencies of the lattice sandwich plate. It can be seen that all the natural frequencies increase with the cross-sectional size of the struts increasing. Fig. 4.4 shows that with the height of the sandwich cores increasing, the natural frequencies first increase and decrease later. And the maximum values appear when the height of the sandwich cores are 0.04m. So by changing the thickness of the face-sheets, the height of the sandwich cores and the cross-sectional size of the struts, proper natural frequencies of the double-deck sandwich plate could be obtained to satisfy the practical requirements.

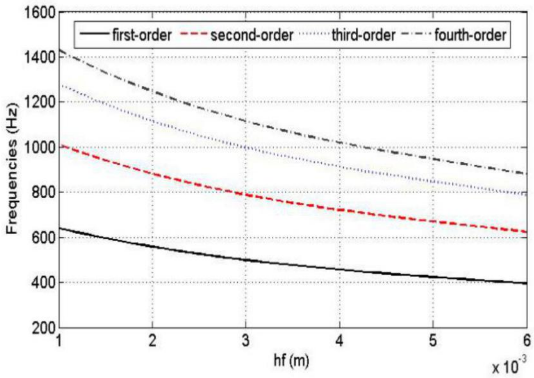

Fig. 4.1

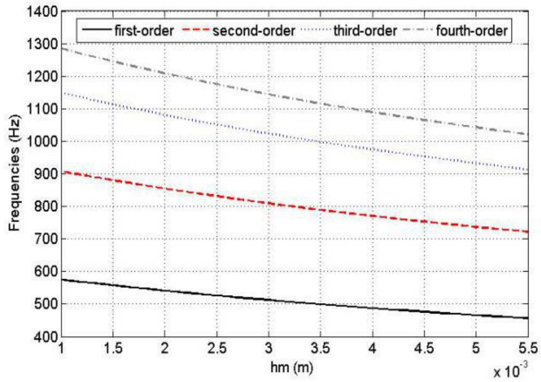

Fig. 4.2

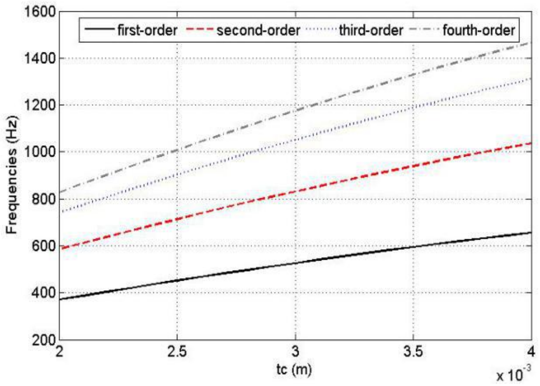

Fig. 4.3

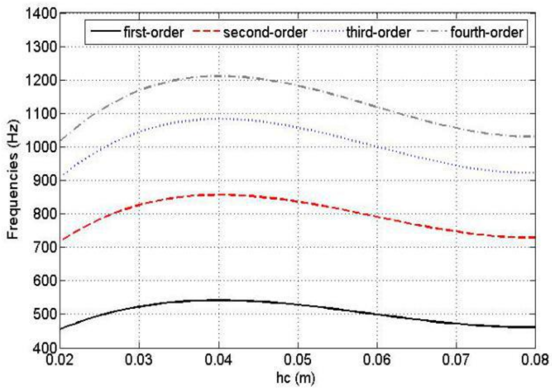

Fig. 4.4

4. Conclusion

An effective methodology is proposed to analyze the free vibration of the simply supported double-deck sandwich plate with lattice truss core. And the effects of the structural properties on the vibration characteristics have been analyzed. From the investigations, the following conclusions can be drawn:

(1) The natural frequencies of the double-deck lattice sandwich plates can be calculated by the present method.

(2) The structural parameters have different influences on the natural frequencies of the lattice sandwich plates. With the thickness of the upper, lower and middle face-sheets increasing, the first four natural frequencies decrease. And the natural frequencies increase with the cross-sectional size of the struts increasing. As the height of the sandwich cores increase, the natural frequencies first increase and decrease later.

APPENDIX

$$A=\lambda(h_1^2h_2+h_4h_5^2+h_1h_2h_3+h_3h_4h_5)+\lambda^2(h_1h_2^2+h_4^2h_5)+\frac{h_1^3+h_5^3}{3}+\frac{h_1^2h_3+h_3h_5^2}{2}+\frac{h_1h_3^2+h_3^2h_5}{4}+\frac{h_3^3}{12}$$

$$B=\rho(\frac{h_1^3}{3}+h_1^2h_2+\frac{h_1^2h_3}{2}+h_1h_2^2+h_1h_2h_3+\frac{h_1h_3^2}{4})+(1-\lambda)\rho h_1h_2(h_1+2h_2+h_3)+(1-\lambda)^2\rho h_1h_2^2+\frac{\lambda^2\rho_c}{3}(h_2^3$$

$$+\frac{3h_2^2h_3}{2}+\frac{3h_2h_3^2}{4})+\frac{\lambda(1-\lambda)\rho_ch_3}{2}(h_2^2+h_2h_3)+\frac{(1-\lambda)^2\rho_ch_2h_3^2}{4}+\frac{\rho h_3^3}{12}+\frac{\lambda^2\rho_c}{3}(h_4^3+\frac{3h_3h_4^2}{2}+\frac{3h_3^2h_4}{4})$$

$$+\frac{\lambda(1-\lambda)\rho_ch_3}{2}(h_3h_4+h_4^2)+\frac{(1-\lambda)^2\rho_ch_3^2h_4}{4}+\rho(\frac{h_5^3}{3}+h_4h_5^2+\frac{h_3h_5^2}{2}+h_4^2h_5+h_3h_4h_5+\frac{h_3^2h_5}{4})+(1-\lambda)^2\rho h_4^2h_5$$

$$-(1-\lambda)\rho h_4h_5(h_3+2h_4+h_5)$$

REFERENCES

1 Reddy J N, Kuppusamy T. Natural vibrations of laminated anisotropic plates. *Journal of Sound and Vibration*, 94(3), 63-69, (1984).

2 Reddy J. N., Phan N. D. Stability and vibration of isotropic and laminated plates according to a higher-order shear deformation theory. *Journal of Sound and Vibration*, 98(2), 157-170, (1985).

3 Wicks N., Hutchinson J. W. Optimal truss plates. *International Journal of Solids and Structures*, 38: 5165-5183, (2001).

4 Hwu C., Chang W. C., Gai H. S. Vibration suppression of composite sandwich beams. *Journal of Sound and Vibration*, 272: 1-2, (2004).

5 Lok T. S., Cheng Q. H. Free vibration of clamped orthotropic sandwich panel. *Journal of Sound and Vibration*, 229(2): 311-327, (2000).

6 Liu T., Deng Z. C., Lu T J. Structural modeling of sandwich structures with lightweight cellular cores. *Acta Mechanica Sinica*, 23(5): 545-559, (2007).

7 Xu M. H., Qiu Z. P. Free vibration analysis and optimization of composite lattice truss core sandwich beams with interval parameters. *Composite Structures*, 106: 85-95, (2013).

8 Chen J. W., Liu W., Su X. Y. Vibration and buckling of truss core sandwich plates on an elastic foundation subjected to biaxial in-plane loads. *Computers, Materials and Continua*, 24(2): 163-181, (2001).