

ULTRASONICS IN INDUSTRY SESSION

## The Graphical Analysis of Ultrasonic Transducers

by Z. Jagodzinski

The dynamic parameters of an ultrasonic transducer may be obtained from its motional immittance diagram (1,2). It may be expressed in either impedance or admittance terms (fig.1) and it consists of the motional circle and electrical line (a constant electrical  $Q_e$  near resonance is assumed). Hence the dynamic parameters of the transducer may be found, i.e. the resonance frequency  $f_0$ , the electromechanical coupling coefficient  $k$  and the quality factors of the mechanical and electrical branches  $Q_m$  and  $Q_e$ . The power transfer factor (3)

$$n = k^2 Q_m Q_e \quad 1$$

which seems to be a very informative and convenient quantity to determine the operational data (efficiency, band-width, loading conditions etc.) may be easily found as

$$n = \frac{R_m}{r_c} \quad \text{or} \quad n = \frac{G_m}{g_e} \quad 2$$

The impedance of the transducer is the vector sum of the impedances of the electrical and mechanical branches:

$$Z_t = Z_e + Z_m \quad 3$$

and it corresponds to the parallel - series equivalent circuit (fig.2). The imaginary part of the electrical line  $X_e$ , is inductive in character in piezomagnetic transducers, and capacitive in character in the piezoelectric ones. Similarly, when the admittance is measured:

$$Y_t = Y_e + Y_m \quad 4$$

and this corresponds to the series - parallel equivalent circuit (fig.3) which is generally accepted for piezoelectric transducers.

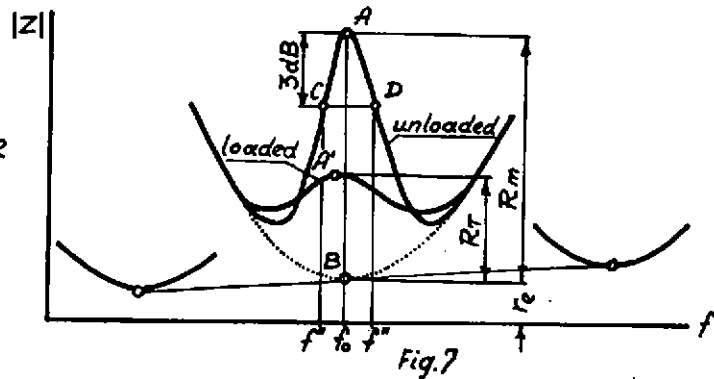
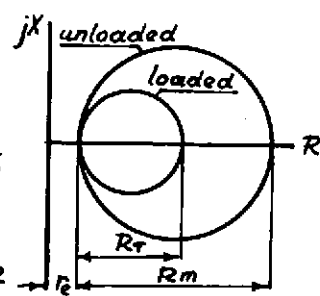
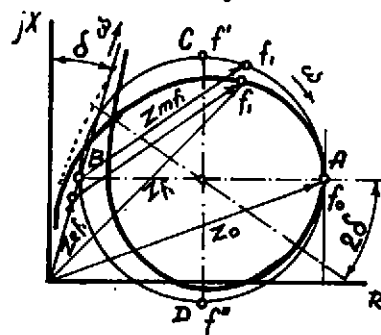
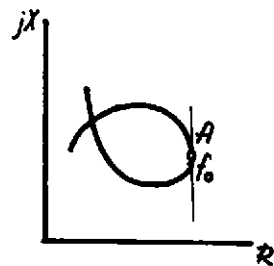
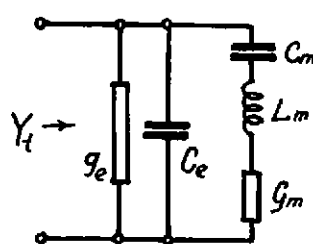
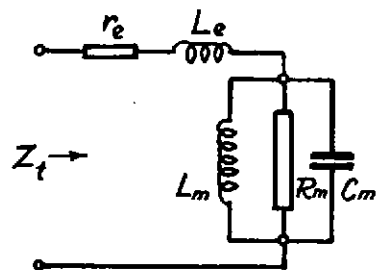
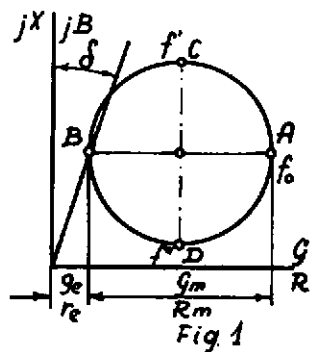
Obviously when the impedance is measured, the admittance may be readily calculated as:

$$Y_t = \frac{1}{Z_t} \quad 5$$

which means that the parallel-series equivalent circuit has been transformed into its dual, series-parallel form and the resonance and anti resonance frequencies exchange their position on the frequency scale (4).

The measurement of the complex immittance loop is, however, a tedious and time consuming work unless a rather refined, automatic impedance plotter is used and the result is not the motional circle but a composite loop which may be considerably distorted (fig.4) especially when the transducer is loaded.

It is then necessary to split the loop into circle and line and this must be done not by "cut and try" drawing, but by a suitably accurate graphical process. The true shape of the



loop may be found when the phasors of the electrical and mechanical branch are added for corresponding frequencies. This is shown on fig.5 using only the impedance terms for simplicity. The loop is "pulled" inside the motional circle and its distortion depends on frequency scale along the electrical line, i.e. on the  $L_c$  and  $R_m$  values (or coupling coefficient  $k$ ).

That is why the loop is more distorted when the transducer is loaded. When it is unloaded the loop usually runs very closely to the "pure" circle.

The point A, corresponding to the resonance frequency of the mechanical branch  $f_0$ , is one, and the only one point of the loop that remains on the circle periphery.

The loop is rotated at an angle about  $2\delta$  where

$$\cotg \delta = Q_e \quad 6$$

and it is not asymptotic to the "clamped impedance" line (dotted) as it is usually assumed, but cuts the electric line somewhere below the mechanical branch resonance.

### Conclusion

It should be noticed that to determine the dynamic parameters of the transducer four points only: A,B,C,D are needed.

The tedious work of measuring the immittance loop may be avoided when the immittance modulus is measured with the reactive part of the electrical branch compensated for, with a capacitor in series with the piezomagnetic transducer or with an inductance in parallel with a piezoelectric one. When the electrical branch is thus tuned to resonance, the motional circle or loop is brought down so that its resonance point A is on the real axis (fig.6).

All the necessary point A,B,C,D may be found here, the dynamic parameters may be determined or motional circle may be drawn.

When the compensated modulus function had been measured for unloaded and loaded transducer, the electro-acoustical efficiency may be calculated (3) as:

$$\eta_{ea} = \frac{n}{n+q+1} \frac{q-1}{q}$$

where  $q = \frac{R_m}{R_e}$  is the ratio of the motional circle diameter of the unloaded and loaded transducer.

### Acknowledgements:

The author wishes to thank Professor J.W.R. Griffiths and Dr. J. Szilard for their hospitality, valuable discussions and help in preparing this paper.

### References:

1. Huefer T.F., Bolt R.H.: Sonics. F. Wiley and Sons, New York, 1966.
2. Hunt F.: Electroacoustics. Harvard University Press, 1954.
3. Jagodzinski Z.: Radiation efficiency of ultrasonic transducers. *Archiwum Akustyki* (Warsaw) No.2, 1972.
4. Jagodzinski Z.: On the adequacy of equivalent circuits for piezomagnetic transducers. *Acustica* 21, 1969.