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FINITE ELEMENT ANALYSIS OF ACOUSTIC POWER RADIATED BY ELECTRICAL MACHINES

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1. INTRODUCTION

A knowledge of the relationship between the vibration and noise of an electrical machine is critical to the calculation of its acoustic power [1]. However analytical methods for predicting the acoustic radiation are applicable only to the idealised models having a regular shape, eg. spherical or cylindrical, and cannot account for the effect of end-shields or cater for all topologies of motor, eg. axial-field or square frame. Even finite length cylindrical motors cannot be solved analytically unless they have infinite stiffening baffles at each end [2].

Whilst the finite element method has been applied extensively to analyse the modes of acoustic cavities, comparatively few papers have been published on its application to acoustic radiation [3]. Furthermore, it is usually restricted to the radiator of mode order zero or one (extensive or impulse type vibration). The main difficulty in the application of finite elements to acoustic radiation is in dealing with the far-field boundaries, which must be well represented otherwise the accuracy is very poor. This arises from the fact that the sound pressure level is low at the far-field boundary, whilst the acoustic power radiated by a vibrating structure, such as an electrical machine, should be independent of the enclosing surface used for the calculation. Whilst some authors use the finite element method in the near-field and an analytical method in the far-field, both being combined to solve for the radiated sound pressure distribution [4], others use the finite element in the near-field and the infinite element in the far-field [5][6]. Nevertheless, limitations still exist with the application of finite elements to the analysis of high frequency and/or large acoustic radiation fields, such as emanate from electrical machines.

The boundary element method provides a basic solution which inherently accounts for the characteristics of the far-field, and makes it unnecessary to deal with the far-field boundaries, and also reduces the field dimension by one. However, whilst these two advantages should make the method ideal for solving acoustic radiation problems, at the natural frequencies (or characteristic frequencies) of the related interior field the equations obtained by the ordinary boundary element lead to ill-conditioned equations or a non-unique solution. Special boundary element methods must then be used [7][8].

In this paper, the finite element method is used to study the acoustic power radiated by an axisymmetric electrical machine. It is combined with a Fourier series analysis, the Fourier series being used to represent the r-Θ variation of sound pressure, whilst finite elements are used to interpolate and discretise the variation in the r-z plane. The combination of both methods allows the acoustic radiation of electrical machines to be predicted with due account of the effect of end-shields, which would not be possible with existing analytical methods. Predictions of acoustic power are validated against measurements on a 3-phase induction motor. The technique is then applied to the spherical model of acoustic radiation.

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2. CALCULATION OF ACOUSTIC POWER USING FINITE ELEMENTS

It is well known that the noise which results from electromagnetically induced vibrations consists not only of an axisymmetric component (circumferential mode order $n = 0$), but also high order harmonics ($n > 0$), whose sound pressure varies with circumferential angular position. In electrical machines the latter is the most important, because i) the vibration produced by a harmonic airgap field usually has a comparatively higher spatial mode order due to the presence of slots and teeth, ii) the natural frequencies of the stator for a circumferential mode of order zero are usually higher than those for higher order modes. However, neither planar or axisymmetric finite elements can solve the acoustic field if the sound pressure varies around the circumference. To cater for this a 3-d representation is necessary. However a 3-d element analysis would be impractical because of the necessarily large number of elements which would be required to model the infinite free-field and the high vibration frequency of electrical machines. The combination of finite element and Fourier series analyses on the other hand, as proposed in the paper, provides a feasible approach to the calculation of the acoustic power radiated by an electrical machine, since it is based on the:

- Finite element discretisation of an r-z plane of a machine, and
- Fourier series representation of the circumferential variation of sound pressure radiated by the machine, as indicated in Fig 1.

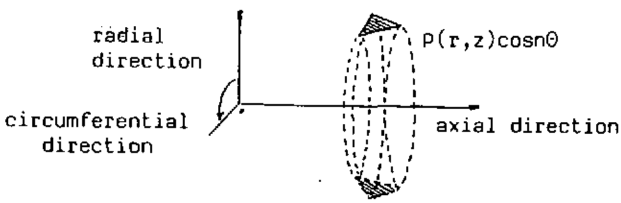

Fig 1 Illustration of Element

The main advantage of the technique is that the dimension of the field is reduced by one, so that the 3-d field is calculated by a 2-d method.

For a harmonic axisymmetric field, the Helmholtz equation and relevant boundary conditions are:

$$\left\{ \begin{array}{ll} \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p}{\partial \Theta^2} + \frac{\partial^2 p}{\partial z^2} + k^2 p = f, \text{ in } V & \\ p = \sum \tilde{p}_o (r,z) \cos n\Theta \, e^{-j\omega t}, & \text{on } \Omega_1 \\ \frac{\partial p}{\partial n} + jkh_n p = j\rho ck \sum V_{no} g_n \cos n\Theta \, e^{-j\omega t}, & \text{on } \Omega_2 \end{array} \right. \quad (1)$$

where $\sum \equiv \sum_{n=0}^{\infty}$

letting $V_n = \sum V_{no} \cos n\Theta \, e^{-j\omega t}$ $\quad\quad f = \sum f_o(r,z) \cos n\Theta \, e^{-j\omega t}$

and

$$p\,(r,\Theta,z,t) = \sum p_o\,(r,z) \cos n\Theta \, e^{-j\omega t} \quad (2)$$

where, for simplicity, the term $\sin n\theta$ in the Fourier series has been neglected since it has no effect on the calculation of acoustic power.

$$\therefore \begin{cases} \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p_o}{\partial r}\right)+\frac{\partial^2 p_o}{\partial z^2}+\left[k^2-\left(\frac{n}{r}\right)^2\right]p_o=f_o, & \text{in } s \\ p_o=\bar{p}_o & \text{, on } \Gamma_1 \\ \frac{\partial p_o}{\partial n}+jkh_n\,p_o=j\rho ck\,V_{no}g_n & \text{, on } \Gamma_2 \end{cases} \tag{3}$$

where the surface S, and its boundaries Γ_1 and Γ_2 are formed by the intersection between region V, and its boundaries Ω_1 and Ω_2 and the r-z plane respectively; $h_n = (1 + \frac{1}{jkR})$; R is the radius of the far-field boundary, $g_n = 1$; V_{no} is the vibrational velocity of the machine; ρc is the sound resistance; $k = \omega/c$; ω is the vibrational angular velocity; and c is the speed of sound.

It can be shown that by letting $k^2 = k^2 - (\frac{n}{r})^2$, the above equations have the same form as those for an axisymmetric formulation. In other words, the harmonic axisymmetric field has been transformed into the ordinary axisymmetric field. Letting $x \equiv z$ and $y \equiv r$ gives

$$dv = 2\pi y ds;\ \text{and}\ d\Omega = 2\pi y dr \tag{4}$$

where $y = \sum_{r=1}^{n'} N_i Y_i$ is the shape function, n' is the node number of an element. Hence the element matrices are given by

$$K_{ij}^e = \int_{se}\left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x}+\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y}\right)2\pi y ds, \qquad C_{ij}^e = \int_{\Gamma_{2e}} h_i N_i N_j\, 2\pi y dr$$

$$M_{1ij}^e = \int_{se} N_i N_j\, 2\pi y ds \qquad M_{2ij}^e = \int_{se} N_i N_j \left(\frac{1}{y}\right)^2 2\pi y ds$$

$$F_i^e = \int_{se} f_o N_i\, 2\pi y ds \qquad G_i^e = \int_{\Gamma_{2e}} V_{no} g_n N_i\, 2\pi y dr \tag{5}$$

and the corresponding assembled equations are:

$$\{[K] + n^2[M_2] - k^2[M_1] + j\,k\,[c]\}\{p_o\} = \{F\} + j\rho ck\,\{G\} \tag{6}$$

It will be noted that compared to an axisymmetric formulation the equations have an extra term, $n^2[M_2]$, which results from the circumferential vibration mode order n.

The acoustic power radiation is calculated from the vibration of the surface of the machine and the sound pressure on the surface [9], viz:

$$W = \sum_{e=1}^{N} W_e$$

$$W_e = \tfrac{1}{2}\,\mathrm{Re}\int_0^{2\pi}\int_{se} \rho V_n^* y dr = \frac{E_n}{2}\,\mathrm{Re}\int_{se} p_o V_{no}^*\, 2\pi y dr \tag{7}$$

where $\varepsilon_n = \begin{cases} 1 & n = 0 \\ 2 & n \neq 0 \end{cases}$

and N is the total number of elements on the machine surface.

The reference acoustic power and the relative sound intensity coefficient are also calculated according to the analytical method given in [9].

For triangular elements with shape function

$$N_i = \frac{1}{2\Delta e}(a_i + b_i x + c_i y) \qquad i = 1,2,3 \tag{8}$$

where $a_1 = x_2y_3 - x_3y_2$ $\quad a_2 = x_3y_1 - x_1y_3$ $\quad a_3 = x_1y_2 - x_2y_1$
$b_1 = x_2 - y_3$ $\quad b_2 = y_3 - y_1$ $\quad b_3 = x_1 - y_2$
$c_1 = x_3 - x_2$ $\quad c_2 = x_1 - x_3$ $\quad c_3 = x_2 - x_1$

Δ^e is the area of the triangular element "e".

$$\therefore \frac{\partial N_i}{\partial x} = \frac{1}{2\Delta^e} b_i \qquad \frac{\partial N_i}{\partial y} = \frac{1}{2\Delta^e} c_i \qquad i = 1,2,3 \tag{9}$$

Meanwhile the acoustic power radiated by a machine can be calculated in the free-field just as if it was being measured in an anechoic chamber. To represent the fact that there is no other sound source in the field, f and {F} are set to zero in the previous equations. The element matrices and the formulae of acoustic power radiated are then deduced and given by:

$$[K^e] = \frac{\pi y^e}{\Delta^e}\begin{bmatrix} b_1b_1 + c_1c_1 & b_1b_2 + c_1c_2 & b_1b_3 + c_1c_3 \\ b_2b_1 + c_2c_1 & b_2b_2 + c_2c_2 & b_2b_3 + c_2c_2 \\ b_3b_1 + c_3c_1 & b_3b_2 + c_3c_2 & b_3b_3 + c_3c_3 \end{bmatrix}$$

$$[M^e]\ \frac{\pi\bar{y}^e\Delta^e}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \qquad [M_2^{\ e}] = \frac{1}{y^{e2}}\ [M_1^{\ e}]$$

$$[c^e] = h_n \frac{2\pi S_{23}}{12}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3y_2 + y_3 & y_2 + y_3 \\ 0 & y_2 + y_3 & y_2 + 3y_3 \end{bmatrix} \quad [G^e] = V_n g_n \frac{2\pi S_{23}}{6}\begin{bmatrix} 0 \\ 2y_2 + y_3 \\ y_2 + 3y_3 \end{bmatrix} \tag{10}$$

where $y_e = (1/3)(y_1 + y_2 + y_3)$, and S_{23} is the distance between nodes 2 and 3 of an element "e" which are assumed to lie on the surface of the machine.

If over S_{23} the vibrational velocity is assumed to be a constant V_{neo}, the acoustic power is:

$$W = \varepsilon_n \sum_{e=1}^{N} \mathrm{Re}\left\{ - \frac{j\omega\bar{y}^{e\prime} S_{23} V_{neo}^{*}}{4\Delta^e} (A_2{}^{o} p_2 + A_3{}^{o} p_3)\right\} \tag{11}$$

where $\bar{y}^{e'} = \frac{1}{2}(y_2 + y_3)$ $\quad A_2^o = \frac{1}{2}(x_1 b_1 + x_2 b_2 + x_3 b_3)$

$A_3^o = \frac{1}{2}(y_1 c_1 + y_2 c_2 + y_3 c_3)$

If over S_{23} the vibrational velocity varies linearly from V_{ne_2} to V_{ne_3}, the acoustic power is:

$$W = E_n \cdot \sum_{e=1}^{N} Re\left\{ \frac{2\pi \bar{y}^e S_{23}}{4\Delta^e} [V_{ne_2}^* (A'_2 p_2 + A'_3 p_3) + V_{ne_3}^* (B'_2 p_2 + B'_3 p_3)] \right\} \qquad (12)$$

where
$$A'_2 = \tfrac{1}{2} a_2 + b_2 \left(\frac{2x_2 + x_3}{6}\right) + c_2 \left(\frac{2y_2 + y_3}{6}\right)$$

$$A'_3 = \tfrac{1}{2} a_3 + b_3 \left(\frac{2x_2 + x_3}{6}\right) + c_3 \left(\frac{2y_2 + y_3}{6}\right)$$

$$B'_2 = \tfrac{1}{2} a_2 + b_2 \left(\frac{x_2 + 2x_3}{6}\right) + c_2 \left(\frac{y_2 + 2y_3}{6}\right)$$

$$B'_3 = \tfrac{1}{2} a_3 + b_3 \left(\frac{x_2 + 2x_3}{6}\right) + c_2 \left(\frac{y_2 + 2y_3}{6}\right)$$

3. EXPERIMENTAL VERIFICATIONS

As mentioned earlier, the far-field boundary has an effect on the calculation of the acoustic power. However, the investigation has shown that, in general, the far-field boundary for electrical machines can be an enclosing spherical surface of 7.5 x the outer radius of the machine and on which a zero-order spherical wave boundary condition can be imposed in the finite element calculation.

Table 1 Comparison between predicted and measured results

	L_{W550Hz}	$L_{W1750Hz}$	L_{WA} (calculated from L_{W550Hz}, $L_{W1750Hz}$)	L_{WA} (direct measurement)
Amplitude of acceleration (m/s^2)	1.70	7.76		
Measured results (dB)	64.3	68.2	69.85	70.6
Calculated results (dB)	60.6	68.3	69.0	

Table 1 shows a comparison between the results calculated by finite elements and measurements on a 3-phase, 4-pole induction motor whose main parameters were: FL rating, 0.8kW, 1380 rpm; encased type stator, outside diameter/axial length of the stator = 0.14m/0.174m; stator/rotor slot number = 24/22. The vibrational acceleration was the average value of measured results at 24 points uniformly distributed over the stator surface. The acoustic power was measured on no-load in an anechoic chamber by measuring the sound pressure at 8 points over an imaginary hemispherical surface of radius 1 metre enclosing the machine. Vibration measurements on the stator and end-shields has shown that the vibration of the end-shields is negligible compared with that of the stator. Therefore in the finite element calculation the vibration of the end-shields is set to zero, although there is no assumption of infinite cylindrical stiffening baffles at the end-shields as is usually the case in analytical cylindrical models of acoustic radiation. The machine exhibits two dominant components, of frequencies 550 Hz and 1750 Hz - which were larger than other components by more than 13dB. Therefore, the vibration and noise spectrum of this machine can be considered as consisting only of these two components, which was confirmed further by comparing the measured acoustic power with/without other components included, Table 1, which showed a difference of < 1dB. By measuring the sound pressure distribution and analysing the electromagnetic and the mechanical noise spectras, as well as by identifying the components from the spectra before/after the power supply was switched off, it was shown that the 1750 Hz component is pure electromagnetic noise, whilst the 550 Hz component is composed of both electromagnetic noise as well as mechanical noise from the ball bearings. The mode orders of the 550 Hz and 1750 Hz electromagnetic components are two, and are produced by the interaction of slot harmonics. The 550 Hz component was dominated by noise produced by the ball bearings and exhibited a relatively unstable amplitude on a spectrum display of a real-time signal analyser. Therefore, the 1750 Hz component was calculated according to the circumferential mode order two, whilst the 550 Hz was calculated according to circumferential mode order zero by assuming the probability of radial vibration along the circumference to be constant. The calculation used 8-node isoparametric elements and 50 divisions along the radius between the outer surface of the machine and the far-field boundary - which was a spherical surface of radius 7.5 times the radius of the machine. The calculated relative sound intensity coefficients were $I_{550\,Hz} = 0.305$ and $I_{1750\,Hz} = 0.819$, and the corresponding calculated acoustic powers are shown in Table 1. Good agreement was achieved, especially for the 1750 Hz component. The 550 Hz component results confirm that the assumption of circumferential mode order zero for the ball bearing noise was quite acceptable. It should be noted that the acoustic power measurements were from the sound pressure, and thus do not consider the phase difference between the sound pressure and the particle vibrational velocity. Therefore the measured results in Table 1 are only reference values. Further comparisons would require more accurate measurements, of the sound intensity, for example.

4. INVESTIGATION OF SPHERICAL MODEL

The vibration of an electrical machine can be approached from that of a sphere [10]. Therefore the radiated sound waves can be approximated by spherical waves radiated by a vibrating sphere. By expressing the vibrational velocity of the stator of the machine as a Legendre series

$$V(\Theta) = \sum_{n=0}^{\infty} V_n P_n (\cos \Theta)\, e^{-j\omega t} \quad r = a \tag{13}$$

where V is the vibrational velocity, V_o is its amplitude, n is the mode order, P_n is a Legendre function, a is the outside radius. The relative sound intensity coefficients are obtained, by solving Helmholtz equation, as:

$$\bar{I}_b = \frac{1}{(ka)^2} \frac{1}{j_{n'}'^2(ka) + n_n'^2(ka)} \quad (14)$$

where j_n' and n_n' are the differential of the first and second spherical Bessel functions. Therefore if the vibration of a machine is known, the radiated acoustic power can be calculated from the relative sound intensity.

$$W = \frac{2}{2n+1} \rho c \pi^2 V_n^2 \bar{I}_b \quad (15)$$

This method has the following features: (i) The model is suitable only for analysing the noise of electrical machine having a length-to-diameter ratio approaching unity, (ii) The vibration produced by the machine must be expressed in the form of Legendre series, (iii) The reference for the relative sound intensity coefficient is not the acoustic power of a planar wave when the vibration is expressed in the form of Fourier series, which is the usual convention, but the acoustic power calculated from the sound wave with its sound pressure and particle velocity in phase and expressed in the form of Legendre series.

However, since the vibrations and hence the radiated noise are caused by electromagnetic forces produced by harmonic fields in the airgap it is much easier to employ Fourier series to describe both the airgap field distribution and the radially excited vibrational forces, and hence the vibration of the stator. Comparing the general forms of Fourier ($\cos n\Theta$) and Legendre ($P_n(\cos\Theta)$) series, they are different except for n=0 and n=1. Therefore in order to predict noise it is necessary to convert the Fourier series into the form of a Legendre series, a difficult and sometimes impossible task.

For the above reasons, many papers employ the relative sound intensity coefficient directly but use the vibration expressed in the form of Fourier series to calculate the radiated acoustic power.

$$W = \tfrac{1}{2} \rho c V_n^2 I_b S_a$$

where S_a is the surface area of the machine.

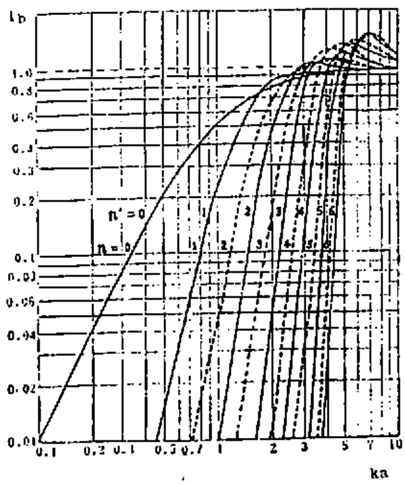

Fig 2 : Relative sound intensity coefficients of spherical model.

When n ≥ 2 dashed line is the results obtained in the paper (vibration expressed in Fourier series), solid line is Jordan's results (vibration expressed in Legendre series) when n = 0 or 1, both are identical.

As a consequence errors are introduced because of the mismatching between the model and the calculation method. In order to highlight these the finite element was applied to the spherical model of acoustic radiation, which is still widely used to analyse the noise of electrical machines having a length-to-diameter ratio of 1.0.

Fig 2 compares the calculated relative sound intensity coefficients of the spherical model obtained from finite element analysis (I_{b_2}) when the vibration of the machine is expressed in the form of a Fourier series with Jordan's results (I_{b_1}) [10]. It shows that when the mode order is 0 or 1, the relative sound intensity coefficients from the two methods are identical, whilst for higher mode orders $I_{b_2} > 1_{b_1}$. Hence the acoustic power from I_{b_1} < that calculated from I_{b_2}, if the same reference is used.

5. CONCLUSIONS

A technique has been presented which combines finite element and Fourier series analyses to calculate the acoustic power radiated from an electrical machine, for which predictions are validated against measurements on a 3-ph induction motor. The technique is capable of accounting for the effect of end-shields on the sound radiation, which would not be possible with existing analytical methods. The application of the finite element method to the spherical model of acoustic radiation has shown that when Jordan's method is used to calculate the radiated acoustic power, it is necessary to express the vibrations of the machine in the form of a Legendre series, otherwise large errors are introduced. The relative sound intensity coefficients of the spherical model, in which the vibration of the machine is expressed in the form of a Fourier series, have been obtained by finite element analysis which makes it possible to use Fourier series analysis throughout.

6. REFERENCES

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